CONTROL SYSTEM DESIGN
TACKLING UNCERTAINTY WITH QFT

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by

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CERTIFICATE

It is certified that the work contained in this thesis entitled "**CONTROL SYSTEM DESIGN : TACKLING UNCERTAINTY WITH QFT**", by "RYALI VENKATARAO", has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Computer-aided control system design, using quantitative feedback theory (QFT), for continuous time, linear time invariant, single-input single-output, systems with parametric uncertainty, has been tackled in this thesis. The general parametric uncertain model assumed here involves a transfer function with its coefficients given as polynomials over a set of system parameters, varying in a real hyper-rectangle. A modified QFT algorithm, based on the extremal, rather than, as is done traditionally, the entire, system frequency response, has been implemented here. The attendant frequency response extrema computation is based, for the general polynomial uncertainty case, on an uniform grid search scheme. For a special case of systems with polynomial uncertainty, viz. the linear affine uncertainty systems, the extremisation is based on a frequency response mapping theorem. This results in a computationally efficient algorithm, relative to the grid method, for the linear affine uncertainty case. The consequent improvement in the computational aspects and the ease of applicability of the basic QFT design paradigm, achieved using the modified QFT algorithm, relative to the traditional QFT approach, is demonstrated through the solving of five design examples.

To my Parents and Sister
with gratitude and affection

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CHAPTER 1

INTRODUCTION

1.1 The Feedback Issue

Technological systems like electronic amplifiers, process plants, air and space craft, ships, etc., are designed to perform in a certain required manner. But it is usually observed that these systems exhibit a behaviour that fluctuates about a desired or nominal behaviour. The reasons for this can essentially be traced to the following factors -

(a) Mathematical model uncertainty - The design process involves the use of mathematical models which for reasons of ease of understanding and applicability, are just approximations to the actual system. The resulting loss of system information renders the system behaviour about the designed for value unpredictable or uncertain.

(b) External factors - The design process also assumes a certain specific set of inputs to the system. However, external factors (also known as noise or disturbances) like e.g. electromagnetic interference, ambient temperature fluctuations, wind speeds, etc., cause the inputs to change randomly about the set values thereby inducing uncertainty in the system output.

Certainly remedial measures would be called for if the system behaviour uncertainty (in the form of output fluctuations) is "too much" i.e. cannot be tolerated. Feedback control is a technique which specifically and effectively addresses this problem viz. that of reduction of system behaviour uncertainty in the face of

model and input uncertainty [HOR63]. Feedback essentially involves monitoring the time flow of the system characteristic to be controlled like e.g. output of a voltage supply, temperature in a chemical reactor, motion of an aircraft or ship, etc., and based on its behaviour rectify the input responsible for this system characteristic like e.g. input voltage, fuel flow, etc. . Monitoring and rectification could involve processes like amplification, transduction, etc. . Setting up these monitoring and rectification sub-systems is what control system design is all about.

1.2 The Feedback Design Problem

A commonly used block diagram representation of the feedback idea is as shown below -

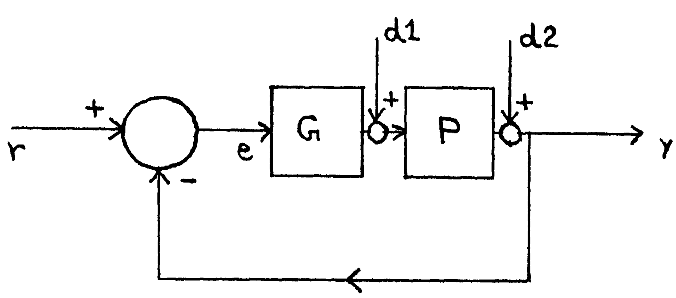


Figure 1.1 The basic feedback configuration

Here, P is the given system (also known as the plant) and "r" is the desired output. The actual system output "y" is compared with "r" and the resulting error "e" (= r - y) is used to drive a corrective element G (also known as a compensator) which in turn drives P in such a manner as to reduce the error "e" to zero.

"d1" and "d2" are external plant input and output disturbances, respectively.

Synthesis of a "low cost" G constitutes the feedback design problem. The cost of G is specified in terms of the complexity of G with regard to its gain and bandwidth. Given a specific problem i.e. a plant P and a set of specifications characterising the output "y", many approaches are available for synthesising G. However, we could possibly classify all the existing design approaches into the following two broad classes -

CLASS 1 Classical approach - This is the approach encountered in most control engineering texts like e.g. [BOW58], [TAK66], [HOR63], [HOS89], [D AZ60], to name a few. Here, design is done for only a single model of P (known as the nominal model). In this method, pioneered essentially by Hendrik Bode [BOD45], uncertainty is indirectly and crudely accounted for through the specification of two design parameters, viz. the gain and phase margins [BOD45], [D'AZ60]. The specification of these margins is, however, highly heuristic without any rigorous analyti-al basis. Thus improper specifications are possible which could lead to a great deal of trial and error (commonly referred to as "tuning the controller") in the design process. This aspect makes the following approach more attractive.

CLASS 2 Modern approach - Here, the model uncertainty, quantified in an appropriate manner, is explicitly

utilised. This feature essentially eliminates a great deal of cut and try that may have to be resorted to, when alternatively, the class 1 approach is used. Mathematically rigourous and quite reliable theories of LQG-LTR [DOY81], H_∞ control [FRA87], μ analysis / synthesis [STE91] and quantitative feedback theory (QFT) [HOR91] fall under this class.

QFT, a frequency domain technique, pioneered by Isaac Horowitz, differs from the other techniques in the way uncertainty is quantified. In the frequency domain, there are mainly two types of model uncertainty, viz. *structured* and *unstructured*. The uncertainty in the system induced by variable system parameters like e.g. variable value passive electrical components, variable value gain factors, variable value inertia elements, etc., is said to be *structured*. This uncertainty makes itself felt essentially in the low frequency range. The uncertainty induced by parasitic elements and unmodelled dynamics, which make themselves felt at high frequencies, is said to be *unstructured*. References in this regard are [DOY81], [KWA93], [HOR91]. QFT utilises the structured uncertainty representation as aga nst the use of the unstructured representation by the other techniques.

As a design technique QFT is quite appealing relative to the other techniques on account of the following reasons -

(1) The structured uncertainty representation is more realistic relative to the unstructured representation [BHA87]. Also the design effort (in terms of the gain and bandwidth of G) is usually exercised over the low

frequency range, over which structured uncertainty is more predominant than the unstructured representation. These factors and the explicit usage of structured uncertainty make QFT well suited to tackle realistic design problems. Though it is possible to handle low frequency structured uncertainty using the other methods, this however, involves conversion of the structured representation into unstructured [LAU86] and in the process could result in overdesign i.e. unnecessary design, or worse still, fail to give a solution for an otherwise QFT solvable problem [BAI91].

(2) In the multi-input, multi-output (MIMO) QFT problem, the issue of guaranteeing specifications for the individual transfer function entries of the closed loop system transfer matrix is directly addressed to. This feature is absent in the other techniques [HOR82].

(3) In the other techniques, synthesis of G is very much dependent on the choice of a nominal model for the system. This necessiates a search for an optimal nominal model which would result in the cheapest controller G. In contrast, QFT is quite insensitive to the choice of a nominal model for the system P. In fact as shown in this thesis, a nominal model need not be chosen at all for the QFT design procedure.

(4) The trade-offs involved between the performance specifications and the cost of G are more transparent in QFT than in the other techniques.

1.3 Scope of the Thesis

Software for handling QFT design for single-input, single-output (SISO) continuous time, linear time invariant (LTI) systems has been developed in this thesis.

The attempt here was to fulfil the need felt for a computer aided QFT design scheme. To date, no commercial software for the same is available [SPEC93]. The reliability of this software was tested by using it to solve five examples. These five examples were so chosen as to cover, as far as possible, a broad spectrum of SISO systems and design problems encountered in real life. They include an unstable system and a nonminimum phase system as well. This software, when used in conjunction with MATLAB (including the CONTROL TOOL BOX) [MAT87], should provide a very easy to use tool for tackling real life feedback system design problems as well as assist in teaching QFT. Further research in QFT can also be facilitated using this software.

1.4 Organisation of the thesis report

This thesis report is organised in the following manner -

Chapter 2 gives a formal mathematical definition of the feedback design problem tackled by QFT.

Classical QFT as pioneered by Marcel Sidi and Isaac Horowitz [HOR72], is described in chapter 3.

Chapter 4 contains a description of the algorithm used in implementing the computer-aided QFT design scheme of this thesis.

The solution for the five examples obtained using this software are presented in chapter 5.

Finally, a summary of the work done and scope for future work

are discussed in chapter 6.

Appendix A contains a description of the software and its use explained by means of an example.

CHAPTER 2

QFT PROBLEM STATEMENT

In this chapter we will give a formal mathematical description of the feedback control system design problem handled in this thesis using QFT.

2.1 System Description

QFT utilises a model for the plant P incorporating the structured uncertainty representation. As mentioned in section 1.2, chapter 1, this type of uncertainty arises essentially due to the fluctuations in system parameter values about their mean or nominal values. More specifically, these system parameter variations give rise to a family of transfer functions given as -

$$\mathcal{P}(s) = \{ P(s,\mathbf{q}), \ \mathbf{q} \in Q\} \qquad 2.1(a)$$

where

$$Q = \{ \mathbf{q} \in \mathbb{R}^n \mid q_i^- \leq q_i \leq q_i^+ , \ 1 \leq i \leq n\} \qquad 2.1(b)$$

where $\mathbf{q}$ is an n-tuple of real uncertain system parameters. Each transfer function $P(s,\mathbf{q})$ belonging to this family can be cons dered to have a form given as -

$$P(s,\mathbf{q}) = \frac{\sum_{i=0}^{m} b_i(\mathbf{q}) \ s^{m-i}}{\sum_{i=0}^{n} a_i(\mathbf{q}) \ s^{n-i}} \qquad (2.2)$$

Thus all the $P(s,\mathbf{q})$ belonging to $\mathcal{P}(s)$ can be considered to be correlated to each other via the functional dependence of the coefficients a_i and b_i on the uncertain parameter vector $\mathbf{q}$.

A set of uncorrelated transfer functions obtained by

linearisation of a nonlinear system model about several operating points, is also sometimes considered to constitute a structured uncertainty model for the system involved [AZV92]. However, in this thesis we were not concerned with this type of structured uncertainty models. Consequently, it should be noted that the software developed in this thesis cannot handle this type of design problems.

2.2 Design Specifications

Feedback control systems can essentially be classified into the following two types [TAK66] -

(1) Tracking control systems - Here the contolled output y(t) (refer figure 1.1) is required to follow or track a randomly varying reference input r(t). Here, the most desirable system property, obviously, is that y(t) should follow the variations in r(t) as rapidly and accurately as possible. Generally used performance measures for this property are -

(i) Rise and settling times of the system unit step response.

(ii) Peak overshoot of the system unit step response.

A generally used single loop tracking system is as shown below -

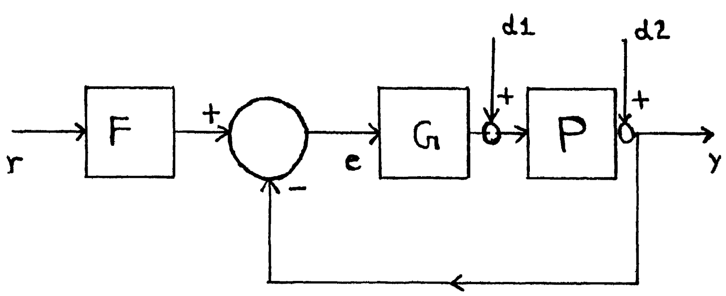


Figure 2.1 The general feedback configuration

This system is the same as that given in figure 1.1, except for the additional feature of a pre-filter F(s). The compensator G(s) reduces the variation in the output y(t) due to system model and inputs uncertainty, while the pre-filter F(s) helps in shaping y(t) to satisfy the tracking specifications.

(2) Regulators - Here the output y(t) is sought to be maintained at a fixed value throughout the operation of the system. Consequently, the reference r(t) is a constant. Here, when the system is subjected to a disturbance, the controlled ouput y(t) should retrieve its original value as quickly as possible and with the least steady state error possible. Also, the resulting transients should be sufficiently damped. Generally used performance measures are -

(i) Settling time and peak overshoot of the system impulse response.

(ii) Steady state error of the system unit step response.

Uncertainty in the system model and inputs, forces us to define a tolerance band for the values assumed by the performance measures mentioned above. For example, for a unit step response we may desire a specification such that

Peak overshoot $M_p \in [M_p^-, M_p^+]$,

Rise time $t_r \in [t_r^-, t_r^+]$, etc. .

Since QFT is a frequency domain technique, these performance specifications,which are in the time domain, have to be first translated to the frequency domain. Unfortunately, however, to date, there is no rigourous technique to carry out this translation. Consequently, a more or less heuristic technique

is adopted for this purpose [AZV92], [D AZ88], [ASH82].

Here, the key lies in the excellent correlation between the time domain and frequency domain responses of a second order system and the fact that a higher order system s response can be approximated by that of a second order system s [TAK66] , [D AZ88].

Two transfer functions, $T_u(s)$ and $T_l(s)$, are found which approximately define the upper and lower bounds, respectively, of the given time domain performance measures. If the closed loop transfer function from r(t) to y(t), viz. $T_{yr}(s)$, is such that

$$0 \leq |T_l(j\omega)| \leq |T_{yr}(j\omega)| \leq |T_u(j\omega)|, \qquad \forall\ \omega \geq 0 \qquad (2.3)$$

then, as a rule of thumb, the time domain performance measures will be satisfied. It should however be noted that this may not be always true. Consequently, some amount of cut and try could be required in effecting the required time domain to frequency domain conversion. To facilitate this trial and error process, two second order transfer functions are chosen as an initial guess for $T_u(s)$ and $T_l(s)$ [HOR72], [AZV92], [D'AZ88], [ASH82]. Simple poles and zeros can then be added to the initial guess to represent the time domain specifications more closely. This process can be understood better via the following example -

EXAMPLE 2.1 - Suppose that we have the following time domain specifications (refer example 5.1, chapter 5) :

SP1 : Peak overshoot M_p of the closed loop unit step response (refer figure 2.1) should be less than 20 %.

SP2 : 2% settling time t_s of the closed loop unit step response should be between 6 and 8 seconds.

For the time domain to frequency domain translation, an initial guess for $T^{u}_{yr}(s)$ and $T^{l}_{yr}(s)$, the "boundary" functions here, was taken to be a second order transfer function of the form given below -

$$Tguess(s) = \frac{\omega_n^2}{s^2 + (2\xi\omega_n)s + \omega_n^2}$$

where ξ = damping factor and ω_n = undamped natural frequency.

Using the formulae,

$$\text{Peak overshoot } (\%) = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\text{Settling time} \approx \frac{4.0}{\xi\omega_n},$$

and MATLAB, the following two boundary models were obtained,

$$T^{u}_{yr}(s) = \frac{1.08}{s^2 + 0.94s + 1.08} \text{, with } M_p = 20\% \text{ and } t_s = 8 \text{ sec}$$

$$T^{l}_{yr}(s) = \frac{1.8}{s^2 + 1.248s + 1.8} \text{, with } M_p = 18\% \text{ and } t_s = 6.2 \text{ sec}$$

such that

$$|T^{l}_{yr}(j\omega)| \le |T^{u}_{yr}(j\omega)| \quad , \quad \forall\omega \ge 0.$$

To increase the spread etween $|T^{u}_{yr}(j\omega)|$ and $|T^{l}_{yr}(j\omega)|$ with increasing frequency, without really compromising the time responses, for reasons made clear in the next chapter (see section 3.3, chapter 3), we added a pole to $T^{l}_{yr}(s)$ and a zero to $T^{u}_{yr}(s)$, resulting in

$$T^{u}_{yr}(j\omega) = \frac{(0.216s + 1.08)}{s^2 + 0.94s + 1.08} \quad ,$$

with M_p = 21% and t_s = 7.8 sec., and

$$T^{l}_{yr}(s) = \frac{1.35}{s^3 + 1.998s^2 + 2.736s + 1.35} ,$$

with $M_p = 0\%$ and $t_s = 6.5$ sec.

A test transfer function,

$$Ttest(s) = \frac{2.16}{s^3 + 2.5s^2 + 3.2s + 2.16} ,$$

was obtained through trial and error aided by MATLAB such that

$$| T^{l}_{yr}(j\omega) | \leq | Ttest(j\omega) | \leq | T^{u}_{yr}(j\omega) |, \quad \forall\ \omega \geq 0.$$

The test transfer function, as expected, had a $M_p = 10\%$ and a settling time of 6.75 seconds. Consequently, we can now assume that if the closed loop transfer function $T_{yr}(s)$ is such that it satisfies equation 2.5(a), with the above given $T^{u}_{yr}(s)$ and $T^{l}_{yr}(s)$ functions then the time domain specifications of SP1 and SP2 will be met. This assumption is more or less vindicated in examples 5.1 and 5.2, chapter 5.

Similar frequency domain specifications of the form given in equation 2.3, but with only the upper bound $T_u(s)$ defined, can be used to specify the system disturbance response [D AZ88], [AZV92].

2.2.1 A Digression

In the QFT literature, e.g. [HOR72], [AZV92], one can find attempts to satisfy time domain specifications of the form -

$$l(t) \leq y(t) \leq u(t), \quad \forall\ t \geq 0, \quad \forall\ q \in Q \qquad (2.4)$$

where $l(t)$ and $u(t)$ define time domain bounds on the desirable system output $y(t)$. But as pointed out in [JAY93], [BAR93], this is quite a tough problem to solve as conversion of the equation 2.4 - type specifications to that in equation

2 3 is quite difficult and highly heuristic with not much theoretical support The methods most commonly suggested are of the sort described in example 2 1 above [ASH82], [HOR72] Also, [ASH82] reports a Monte Carlo determination of the allowable frequency response spread corresponding to equation 2 4, given the knowledge of the model parameters and their variances. [KRI77] is another related reference However, the use of these methods is not widespread

But note that equation 2 4 is essentially a tracking specification Consequently, we feel that it would be much more easier to first convert the equation 2 4 - type specifications into an equivalent set of specifications on the unit step response of the system,e g in terms of bounds on the rise time, peak overshoot, settling time, etc Subsequently, these step response specifications can then be converted into the equation 2 3 - type specifications, relatively easily, as illustrated in example 2 1 above However, this type of a design problem has not been considered in this thesis

2 3 Problem Statement

Summarising the above given discussion, we can now formally state the QFT design problem tackled in this thesis, as follows -

Consider an uncertain continuous time system, with a linear time invariant model of the form given in equation 2 2 and embedded in a feedback configuration of the form given in figure 2 1 The QFT design problem, then, is to obtain fixed (i e with no uncertainty), physically realisable, LTI compensator G(s) and

pre-filter F(s) such that the resulting closed loop system shown in figure 2 1 is stable and the following frequency domain specifications are satisfied -

$$| T^{l}_{yr}(j\omega) | \leq | T_{yr}(j\omega) | \leq | T^{u}_{yr}(j\omega) | \qquad 2\ 5(a)$$

$$| T_{yd1}(j\omega) | \leq | T^{u}_{yd1}(j\omega) | \qquad 2\ 5(b)$$

$$| T_{yd2}(j\omega) | \leq | T^{u}_{yd2}(j\omega) | \qquad 2\ 5(c)$$

$$\forall\ \omega \geq 0,$$

where

$$T_{yr}(s) = \frac{y(s)}{r(s)} = \frac{F(s)L(s)}{1\ 0 + L(s)} \qquad 2\ 6(a)$$

$$T_{yd1}(s) = \frac{y(s)}{d1(s)} = \frac{P(s)}{1\ 0 + L(s)} \qquad 2\ 6(b)$$

$$T_{yd2}(s) = \frac{y(s)}{d2(s)} = \frac{1\ 0}{1\ 0 + L(s)} \qquad 2\ 6(c)$$

where

$L(s) = P(s)G(s)$ is the (open) loop gain

A description of the application of the QFT design technique is what follows in the next chapter

CHAPTER 3

THE QFT TECHNIQUE

In this chapter we will describe the QFT design procedure pioneered by Marcel Sidi and Isaac Horowitz [HOR72]

3 1 Essence of the QFT design procedure

Uncertainty in the system model results in a fuzzy phase - magnitude Bode plot More specifically, at each frequency ω_o the frequency response of the system consists of a set of points in the complex plane, given as -

$$TmP(\omega_o) = \{ P(j\omega_o\mathbf{q}),\ \mathbf{q} \in Q\} \qquad (3\ 1)$$

as shown below -

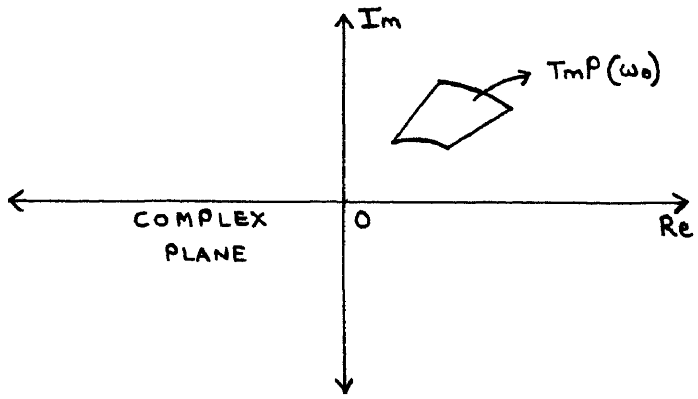


Figure 3 1 Uncertainty template at ω_o

This set is known as a template [HOR72] or a value set [FU90] of the system at the frequency ω_o

Now, the closed loop transfer function from the reference input r(t) to the output y(t) is (refer figure 2.1) given as -

$$Tyr(s) = \frac{F(s)L(s)}{1\ 0 + L(s)} \qquad (3\ 2)$$

where L(s) = P(s)G(s) is the (open) loop gain

Using the log magnitude notation, where

$$Lm[T(j\omega)] = 20\log(|T(j\omega)|) \tag{3 3}$$

with "log" denoting the common logarithm i e to the base 10, we get

$$Lm[T_{yr}(j\omega)] = Lm[F(j\omega)] + Lm[L(j\omega)/(1\ 0 + L(j\omega))]$$

with $$Lm[L(j\omega)] = Lm[P(j\omega)] + Lm[G(j\omega)] \tag{3 4}$$

Uncertainty induced variation in $P(j\omega)$ causes variation in $T_{yr}(j\omega)$ such that -

$$\Delta Lm[T_{yr}(j\omega)] = \Delta Lm[L(j\omega)/(1\ 0 + L(j\omega))]$$

with $$\Delta Lm[L(j\omega)] = \Delta Lm[P(j\omega)] \tag{3 5}$$

Note that G(s) and F(s) are assumed to be fixed (refer section 2 3, chapter 2)

From equations 3 1-3 5 it is quite clear that in the Nichols chart [D AZ88] the loop gain template

$$TmL(\omega_0) = \{P(j\omega_0,\mathbf{q})G(j\omega_0),\ \mathbf{q} \in Q\} \tag{3 6}$$

has the same shape as the plant template $TmP(\omega_0)$, but is translated vertically by $Lm[G(j\omega_0)]$ and horizontally by $\arg[G(j\omega_0)]$, where $\arg[G(j\omega_0)]$ is the phase of $G(j\omega_0)$, as shown below -

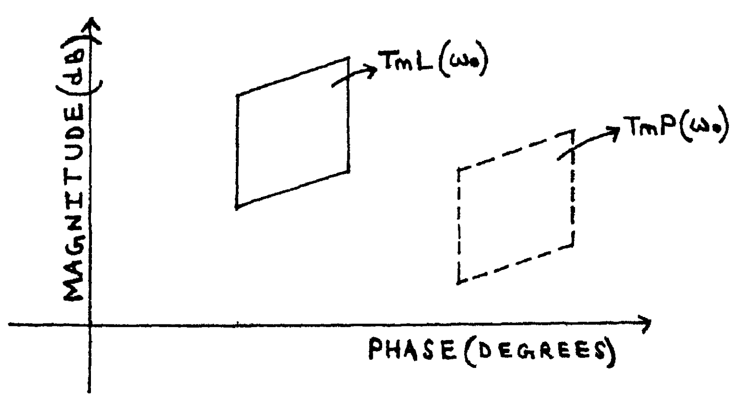


Figure 3 2 Template movement in the Nichols chart

Also, observe that it is in terms of the magnitude deviation of the complementary sensitivity function

$$T_{cs}(s) = L(s)/(1\ 0 + L(s)) \qquad (3\ 7)$$

that the closed loop magnitude deviation is expressed (see equation 3 5) Consequently, given a loop gain template $TmL(\omega_o)$ in the Nichols chart, the corresponding magnitude variation in the closed loop gain at the frequency ω_o can be directly read off the chart via the constant M - contours This aspect and the feature of permanency of the loop gain template shape irrespective of the value of $G(j\omega_o)$ are made use of in the QFT algorithm to be discussed in the course of the following discussion

Now, the specifications 2 5(a) dictate that

$$\Delta Lm[T_{yr}(j\omega)]_{max} \leq Lm[T^{u}_{yr}(j\omega)] - Lm[T^{l}_{yr}(j\omega)], \quad \forall\omega \geq 0 \quad (3\ 8)$$

From equation 3 2 it is quite clear that if $(1\ 0 + L(j\omega_o)) \neq 0, \quad \forall\, q \in Q$, then

$$T_{yr}(j\omega_o) \longrightarrow F(j\omega_o) \quad \text{as} \quad |\,G(j\omega_o)\,| \longrightarrow \infty$$

$$\rightarrow \quad \Delta Lm[T_{yr}(j\omega_o)] \longrightarrow 0$$

In other words, $\Delta Lm[T_{yr}(j\omega_o)]$ can be made arbitrarily small by choosing $G(j\omega_o)$ with $Lm[G(j\omega_o)]$ sufficiently large This essentially means that there exists a $G^*(j\omega_o)$ value such that equation 3 8 will get satisfied for all $G(j\omega_o)$ values having the same phase as $G^*(j\omega_o)$ but satisfying the inequality

$$Lm[G(j\omega_o)] \geq Lm[G^*(j\omega_o)]$$

A formal proof of this statement is given in [BAI88]

Such $G(j\omega_o)$ values will be said to be *admissible performance wise* The design objective is to synthesise such a performance wise admissible compensator $G(s)$ which additionally ensures closed loop stability (i e. is *admissible stability wise*).

This $G(s)$, however, only guarantees satisfaction of equation 3 8 and does not ensure satisfaction of the tracking specifications of equation 2 5(a) This gets done by the pre-filter $F(s)$ For this the pre-filter magnitude at ω_o should be such that

$$Lm[T^{l}_{yr}(j\omega_o)] - Lm[T_{yr}(j\omega_o)]_{min} \le Lm[F(j\omega_o)]$$
$$\le Lm[T^{u}_{yr}(j\omega_o)] - Lm[T_{yr}(j\omega_o)]_{max} \qquad (3.9)$$

Any minimum phase transfer function $F(s)$ which satisfies equation 3 9 for all $\omega_o \ge 0$, would provide a solution for the pre-filter $F(s)$

On similar lines, a performance wise and stability wise $G(s)$ is also sought for the disturbance specifications of equations 2 5(b) and 2 5(c) Note that a pre-filter is not needed for ensuring these specifications

The design procedure, at the very outset, thus requires a set of frequencies, Ω, to be identified such that

$$(1\ 0 + L(j\omega)) \ne 0 , \quad \forall\omega \in \Omega$$

This set is known as the *trial frequencies set* For computational solvability we naturally consider a finite Ω Though the selection of Ω can be arbitrary, a common practice is to include frequencies lying in the DC to ω_h range, where ω_h is a high frequency value such that the structured uncertainty model $P(s)$ (refer equation 2 2) tends asymptotically to k/s^{n-m} (k variable) for all frequencies $\omega > \omega_h$ [D AZ88], [AZV92] The main reason for this choice is that control effort is usually concentrated in this low frequency range

3.2 The Sidi - Horowitz QFT algorithm

This algorithm has the following steps -

STEP 1 Setting up the frequency domain performance specifications - This essentially involves a a conversion of time domain specifications to the frequency domain, essentially on the basis of a semi-heuristic procedure of the kind described in example 2 1, section 2 2 chapter 2

STEP 2 Specification of bounds for a performance admissible $G(s)$ - At the very outset, an appropriate finite set of trial frequencies Ω is selected Then, at each trial frequency ω_o at a fixed assumed phase of $G(j\omega_o)$, viz $\phi(\omega_o)$, a value $G^*(j\omega_o)$, with phase $\phi(\omega_o)$, is found such that for all $G(j\omega_o)$ values with phase $\phi(\omega_o)$ and $|G(j\omega_o)| \geq |G^*(j\omega_o)|$, equation 3 8 or 2 5(b) or 2 5(c) (according to the problem type), is satisfied Several such "bounding" $G^*(j\omega_o)$ values are found by varying $\phi(\omega_o)$ over some desired phase interval of length 360 degrees (e g the interval $[-360^\circ, 0^\circ]$)

As it is the loop gain which is important in the design procedure, these $G^*(j\omega_o)$ values are consequently used to define equivalent lower bounds $L^*(j\omega_o) = P(j\omega_o q)G^*(j\omega_o)$, on a nominal loop gain value $L_o(j\omega_o)$ defined, arbitrarily, for some nominal parameter vector value $q_o \in Q$ Thus, we get several such $L^*(j\omega_o)$ points which can be joined to form a closed continuous curve in the complex plane known as a *B(ω) bound* for the frequency ω_o as shown below -

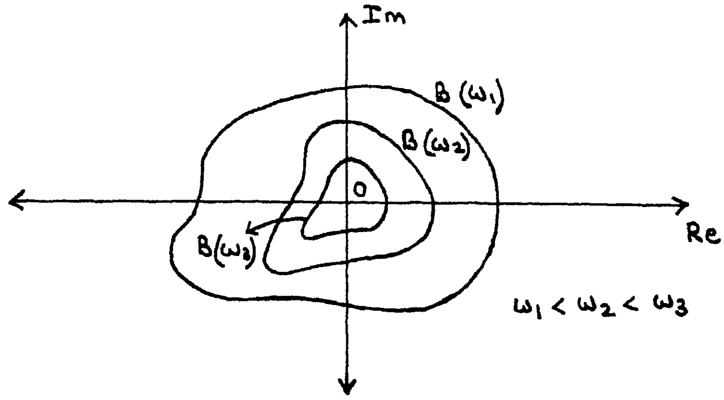


Figure 3 3 The B(ω) bounds

It follows quite easily that for G(s) to be admissible performance wise, the nominal loop gain $L_o(j\omega_o)$should lie above i e outside the corresponding $B(\omega_o)$ bound at each trial frequency ω_o, as shown in the Nichols chart below -

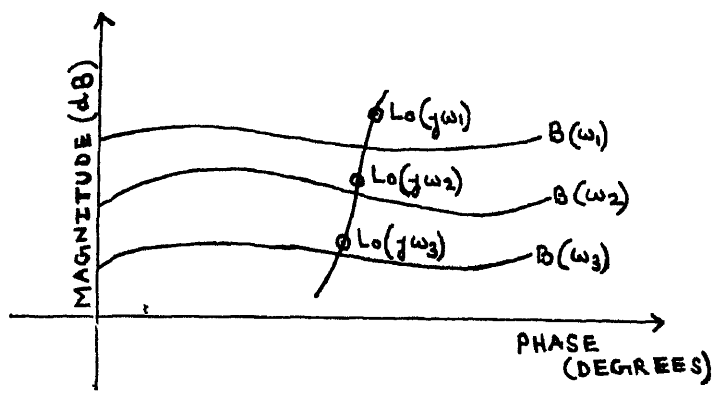


Figure 3 4 Admissible $L_o(j\omega)$ curve

If simultaneous specification of two or all equations 2 5 (section 2 3, chapter 2) is done then for every trial frequency ω_o, we will correspondingly get two or three $B(\omega_o)$ bounds Then we, obviously, take the worst combination of these bounds to get a

composite $B(\omega_o)$ bound, as shown below -

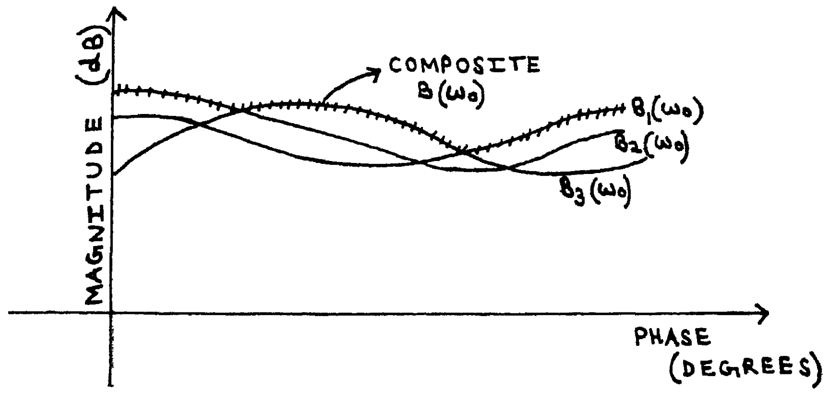


Figure 3 5 The composite $B(\omega)$ bound

The Nichols chart is very convenient in finding these $B(\omega)$ bounds For instance, at each assumed value of the phase of $G(j\omega_o)$, viz $\phi(\omega_o)$, the $L^*(j\omega_o)$ value is found by simply moving the corresponding loop gain template vertically up or down as need be, till, that position is attained where the performance specifications just about get satisfied [HOR72], [D AZ88] The amount of translation (in terms of dB) done in attaining this position will then define the $G^*(j\omega_o)$ value

STEP 3 Loop shaping for stability - In this step, the stability of the closed loop system with the G(s) obtained from step 2 is tested by subjecting the resulting fuzzy open loop polar plot to the Nyquist encirclement test If the test is passed then go to step 4 , else modify G(s) to ensure closed loop stability and go back to step 2 to test for performance admissibility of the modified G(s)

This step involves the usual cut and try of the classical loop shaping method [AZV92] is a very useful reference in this regard

STEP 4 Pre-filter design - If the pre-filter F(s) is not required then end the design procedure, else synthesise a minimum phase transfer function F(s) satisfying equation 3 9 This F(s) will then constitute a solution for the pre-filter and the design procedure can then be ended

Thus QFT is essentially an extension of the traditional loop shaping methodology to design problems with large parametric uncertainty

It will be useful to note that it has been shown [GER80] that an optimal solution for G(s) is the one in which all the $L_o(j\omega_o)$ points lie on their respective $B(\omega_o)$ bounds This solution is optimal in the sense that the resulting $|L(j\omega)|$ will have the fastest possible roll-off rate (as the frequency increases) and in the process, the least sensor noise problem [HOR63], [HOR91]

3.3 Further issues in the design procedure

3 3 1 High frequency behaviour of the $B(\omega)$ bounds

The generation of the $B(\omega)$ bounds, for each trial frequency ω_o, is done by varying $\phi(\omega_o)$ over an interval of length 360° This naturally creates the impression that the $B(\omega)$ bounds for all the trial frequencies, encircle the origin, as shown in figure 3.3 That this may not be

necessarily true (and that in fact it is not desirable) can be explained as follows

For most physical systems, $P(s) \longrightarrow 0$, as $|s| \longrightarrow \infty$ Also, as $G(s)$ is assumed to be proper or strictly proper (the assumption of realisability) we have $L(s) \longrightarrow 0$, as $|s| \longrightarrow \infty$ Consequently, we have

$$T_{yr}(s) \longrightarrow F(s)L(s)\ ,\quad \text{as} \quad |s| \longrightarrow \infty$$

$$\Rightarrow \quad \Delta Lm[T_{yr}(j\omega)] \longrightarrow \Delta Lm[P(j\omega)],\quad \text{as} \quad \omega \longrightarrow \infty$$

This means that as a necessary condition for the satisfaction of the tracking specifications in equation 2 5(a), we must have

$$\Delta Lm[P(j\omega)]_{max} \leq Lm[T^{u}_{yr}(j\omega)] - Lm[T^{l}_{yr}(j\omega)],\ \forall\omega \geq \omega^{*} \qquad (3\ 10)$$

where ω^{*} is a very high frequency

In other words feedback is really not needed for frequencies beyond w^{*} Now, equation 3 10 can be shown to guarantee that as $\omega \longrightarrow \infty$, the $B(\omega)$ bounds start shrinking and beyond ω^{*} cease to encircle the origin but persist in encircling the $-1+j0$ point in the complex plane as shown below [HOR73], [HOR76], [HOR79] -

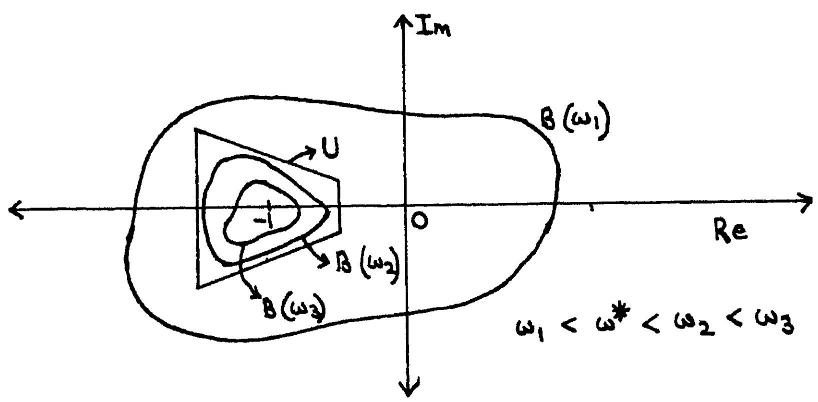


Figure 3 6 High frequency behaviour of the $B(\omega)$ bounds

Consequently, we can define a closed contour U enclosing the $-1+j0$ point but not encircling the origin, as shown in the above given figure, such that for all frequencies $\omega \geq \omega^*$ the corresponding $B(\omega)$ bounds lie entirely inside the closed contour U This contour is known as the universal high frequency (UHF) boundary [HOR72], [D AZ88], as avoidance of U for all $\omega \geq \omega^*$ ensures avoidance of the $B(\omega)$ bounds for all the frequencies above ω^*

The UHF boundary U has an additional significance in that for certain disturbance attenuation problems, a constraint on the damping factor is enforced via an M-contour constraint on the complementary sensitivity function $T_{cs}(s)$ [HOR72], [D AZ88] That is, the performance requirement is to satisfy

$$\left| T_{cs}(j\omega) \right| \leq M \quad , \quad \forall\omega \geq 0, \quad \forall q \in Q.$$

This constraint can be enforced by defining the concerned M-circle as the UHF boundary

The undesirability of persistent encirclement of the origin by the $B(\omega)$ bounds (i e for all ω) can be appreciated due to the fact that it could then imply an unrealisable $G(s)$ To see this, note that persistent $B(\omega)$ encirclement of the origin implies that the nominal loop gain $L_o(s)$ cannot go to zero, asymptotically or otherwise If $P(s,q_o)$ is strictly proper then, to ensure that $L_o(s)$ does not go to zero as $|s| \longrightarrow \infty$, $G(s)$ will have to be improper and hence unrealisable

3 3 2 Variable relative degree of the structured uncertainty model

The relative degree 'e" of P(s) (refer equation 2 2) is the difference between the denominator and the numerator degrees i e e = n - m

A variable relative degree implies the existence of some "disappearing" poles and zeros giving rise to a representation of P(s,q) of the form -

$$P(s,q) = P_1(s,q) \frac{\prod_i (b_i s + 1)}{\prod_i (a_i s + 1)} \qquad (3\ 11)$$

where $P_1(s)$ has a fixed relative degree and each b_i and a_i can take the value 0, thereby giving rise to the phenomenon of disappearing poles and zeros and the consequent variation in the relative degree e [HOR79]

Now, as $P(s,q) \longrightarrow k/s^e$ (k variable), for $|s| \longrightarrow \infty$, if $e(max) \geq 4$, then the loop gain template could have a phase variation of more than 360° and consequently the $B(\omega)$ bounds will tend to encircle the origin as $\omega \longrightarrow \infty$ As explained in the earlier section (section 3 3 1) this is undesirable as it could mean an unrealisable G(s) Consequently, for strictly proper plants, as a necessary condition for the realisability of G(s) e(max) should be limited to 3 [HOR76], [HOR79]

3 3 3 Uncertain unstable plants

Let "np" be the total number of unstable poles of the system P(s) given by equation 2 2 If each unstable pole

remains in the complex right half plane (RHP) over the entire range of uncertainty, then this implies that np is a constant Consequently, for stability, assuming no pole-zero cancellations in P(s)G(s), the fuzzy loop gain polar plot should encircle the -1+j0 point np times in the appropriate sense The fact that np is a constant removes any ambiguity regarding the encirclement count

However, this is not the case when there are certain unstable poles which become stable i e migrate to the left half plane (LHP), for certain parameter values In this case np and hence the Nyquist encirclement count become variables, complicating in the process the stability issue No attempt has been made in this thesis to address this problem of a variable np A problem with a constant np (see example 5 2, chapter 5), however, has been tackled in this thesis.

As an interesting aside, however, the problem of a variable np can be looked upon as a simultaneous stabilisation problem of the following kind -

$$\text{Let} \quad P(s,q) = P_1(s,q)\,\frac{1\ 0}{\prod_i (s - a_i)} \tag{3 12}$$

where $P_1(s\ q)$ is stable and each $a_i \in [a_i^-, a_i^+]$ such that $a_i^- \geq 0$ or $a_i^- < 0$ and $a_i^+ > 0$, i e a_i could be a non-migratory (i.e will always remain in the RHP) or a migratory (i e can cross over to the LHP) unstable pole

Now define the following two sets of systems -

$$P_s(s,q) = P_1(s,q)\,\frac{1\ 0}{\prod_i (s - a_i)} \qquad 3\ 13(a)$$

where $\qquad a_i \in [a_i^-, 0), \qquad a_i^- < 0,$

and

$$P_{us}(s,q) = P_1(s,q) \frac{1\ 0}{\prod_i (s - a_i)} \qquad 3\ 13(b)$$

where $\quad a_i \in [0\ ,\ a_i^+], \qquad a_i^+ \geq 0 \quad ,$ or

$\qquad\qquad a_i \in [a_i^-\ ,\ a_i^+], \qquad a_i^- > 0$

It is fairly obvious that a $G(s)$ which is admissible performance and stability wise, for *both* $P_s(s,q)$ and $P_{us}(s,q)$ with the same set of performance specifications as for the given system $P(s,q)$, will also be a solution for the original problem The point to be noted here is that in this approach as $P_{us}(s,q)$ has a fixed np and $P_s(s,q)$ is anyway stable, the issue of stability can be handled relatively easily, as there will be no ambiguity regarding the Nyquist encirclement count

The problems associated with the computational aspects of the QFT design algorithm described in this chapter and consequent remedial measures adopted, constitute the topic for discussion in the next chapter

CHAPTER 4

ALTERNATIVE QFT ALGORITHM

In this chapter we will discuss some problems associated with the QFT algorithm described in section 3 2, chapter 3 A new algorithm, which takes into account only the extremal open loop behaviour and which forms the basis for the QFT design software developed in this thesis, is described in this chapter

4.1 Alternative QFT design algorithm

The QFT algorithm described in section 3 2, chapter 3, has the following disadvantages

(1) The boundary of the plant template defines the worst case open and closed loop behaviour Consequently, an accurate generation of the template boundary is essential for reliable design Unfortunately, however, accurate and computationally efficient algorithms for generation of the template boundary do not exist, except for some simple cases [BAI88], [BAI89] ,[FU90],[BART93] The most commonly used method is the grid method Here, each uncertain parameter interval is partitioned or gridded in some suitable manner and at each grid point in Q (refer equation 2 1, chapter 2) the system frequency response (magnitude and phase) is computed Unless the grid is extremely fine, this method usually results in an inaccurate definition of the template boundary which can lead to unreliable designs [BART93]

(2) Increase in accuracy of the design procedure can be achieved only with the computation of a large number of values for the $B(\omega)$ bounds (refer sec 3 2, chapter 3) and also for the templates as well The consequent computational labour and memory storage problems could be quite phenomenal and will get further compounded with increase in the number of uncertain parameters This could also possibly force a reduction in the number of trial frequencies that can be considered, thereby impairing the reliability of the design

Now a little contemplation will show that the following points stand out with regard to the computational aspects of this algorithm -

(a) $B(\omega)$ bounds - These bounds are needed only if the Nichols chart is used for the design Having a computer program to check for satisfaction of the performance specifications (equations 2 5, 3 8) should dispense off with the computation of these bounds

(b) Template formation - The actual template, at each trial frequency ω_o, is bounded by the region $ABCD(\omega_o)$, shown in the figure below

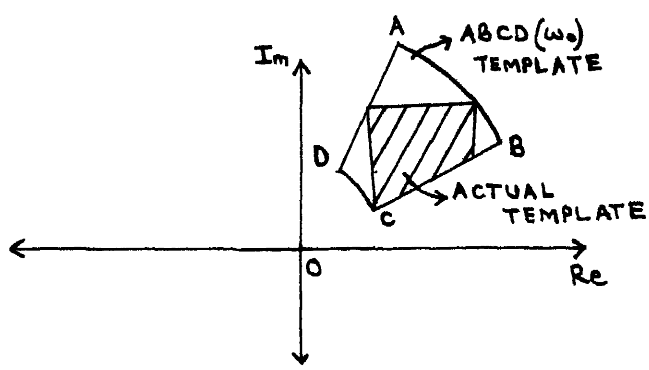


Figure 4 1 The $ABCD(\omega_o)$ template

This region ABCD(ω_o), is obtained by bounding a region in the complex plane by the extrema of the loop gain magnitude and phase It is obvious that a design for the given system, assuming it to have the ABCD(ω_o) type of uncertainty, will hold for the actual uncertainty as well We may thus solve the design problem assuming the ABCD(ω_o) type of uncertainty, dispensing off, in the process, the generation of the actual templates or, for that matter, the entire boundary of each template

On the basis of these observations, we decided to replace the computation of each template (entire or its boundary alone) and the B(ω) bounds by the computation of the following values -

(1) The loop gain extrema (of the magnitude and the phase)

(2) The worst case extrema of the concerned system transfer function (e g $T_{yr}(s)$, $T_{yd_1}(s)$, etc) to check for satisfaction of the performance specifications

This change yields the following advantages -

(1) Only four values, viz the loop gain extrema, have to computed for each template Also, the B(ω) bounds are not needed The resulting savings in computational labour and memory space could be phenomenal

(2) Considerable computational savings made, as mentioned above, also enable a substantially larger number of trial frequencies to be considered, thereby improving the reliability of the design

(3) Explicit computation of the extremal open loop behaviour helps in visualising a rough open loop polar plot without resort to any graphics utility Stability,

can then be quickly determined Thus dependence on graphical utilities, which is considerable in the earlier approach, is greatly reduced here, thereby enhancing the user friendly aspect of any algorithm based on this approach

An improved QFT algorithm based on this approach is as follows

STEP 1 Time domain specifications are converted to the frequency domain on the basis of a semi - heuristic approach described in section 2 2, chapter 2

STEP 2 Define an appropriate, finite set of trial frequencies Ω

Assume some appropriate realisable transfer function as an initial guess for G(s) Preferably, assume G(s) = 1 0, as it will give a rough idea about the amount of gain needed for satisfaction of the performance specifications

STEP 3 Check if G(s) is admissible performance wise at each trial frequency If at some trial frequency ω_o G(s) is inadmissible, then a a bound (lower or upper, depending on the design problem) on the allowable $|G(j\omega_o)|$, is returned Such bounds can be used to define another G(s) candidate and this step is then repeated

STEP 4 Check if G(s) is admissible, stability wise

If it is not, assume another structure for G(s) and go back to step 3

STEP 5 If the pre-filter is not needed, then end the design procedure, else synthesise a minimum phase transfer function F(s) which satisfies equation 3 9 and end the design procedure

The resulting G(s) and, if designed, F(s) will then constitute a solution for the QFT design problem This approach, for want of a better name, is called as the *mini-max approach*

A disadvantage, however, of this design procedure could be overdesign i e unnecessary design For example, assume a minimum phase system with a ABCD(ω_o) type uncertainty at the real axis crossover as shown below

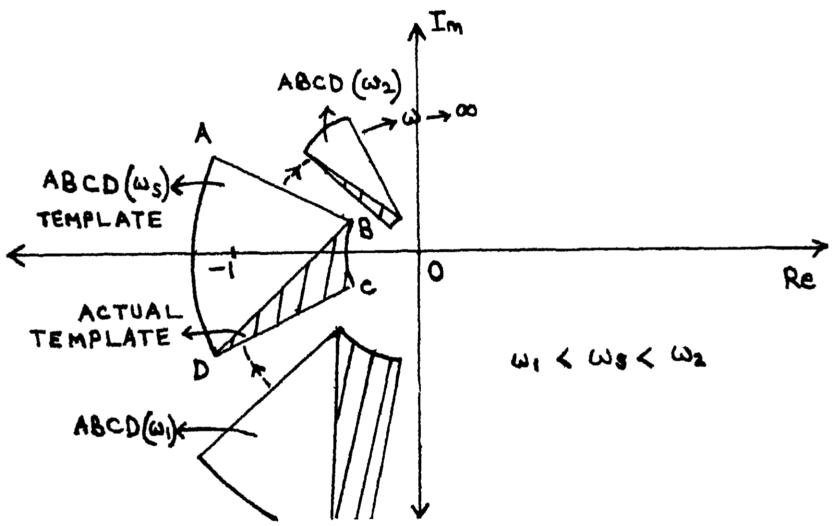


Figure 4 2 Possible overdesign in the mini-max approach

Observe that the actual fuzzy open loop polar plot avoids encircling the -1+j0 point thereby indicating a stable closed loop system However, the ABCD(ω_s) template at the real axis crossover

indicates an unstable system as it includes the $-1+j0$ point. Consequently, based on this erroneous information one might go for lead or lag compensation as deemed fit which, however, in actuality is unnecessary.

But it may be noted that if the width and/or breadth of the ABCD(ω_*) template at the real axis crossover, crucial for stability considerations, is quite small, then the resulting overdesign will also be correspondingly less. The advantages of this approach, mentioned earlier, will then certainly outweigh the disadvantage of overdesign. Consequently, we may, under such conditions, overlook such overdesign.

4.2 Computational problems associated with the mini-max approach

Extremization of the loop gain and the concerned system transfer function (e.g. $T_{yr}(s)$, $T_{yd2}(s)$, etc.) is a very crucial step in the mini-max approach. This problem of extremisation, a non-linear programming problem [RAO84], can be stated mathematically as -

$$\text{Extremise } f(\mathbf{q}), \quad \mathbf{q} \in Q \qquad (4.1)$$

where $f(\mathbf{q})$ could be the magnitude or phase of the specified transfer function and $\mathbf{q}$ is the uncertain parameter vector (refer equation 2.1).

However, the solution of equation 4.1, obtained using any efficient numerical nonlinear programming method is always a local extremum which need not be global over Q [RAO84]. Consequently, there is the danger of ending up with erronous results and hence erronous designs.

This fact forced us to take up the following two

computational alternatives -

(1) One alternative is to carry out a blind search over the entire domain of Q for the extrema Computationally, it means that we arbitrarily partition Q into a grid gr(Q) and at each grid point $q^* \in gr(Q)$, compute $f(q^*)$ A straight forward search among the resulting $f(q^*)$ values will then give an approximation to the actual extrema The accuracy of this method, obviously, will depend on the fineness of the grid gr(Q) Increase in accuracy however, can be achieved only with an increase in computational labour This method, known as the grid method, also becomes computationally inefficient and/or intensive with increase in the number of uncertain parameters This method, however, at the worst, is as inefficient as the earlier given algorithm (section 3 3, chapter 3)

A mini-max algorithm based on the grid method with uniform partitioning of the grid has been implemented in this thesis

(2) Another alternative is based on an *a priori* knowledge of a small region R in Q, where the extrema are guaranteed to be located The region R should be small enough so as to search and locate the extrema with the minimum computational labour and maximal accuracy possible Powerful optimisation routines like those found in the NAG or IMSL math libraries can be utilised for this purpose This method, if possible to apply, would be more efficient and faster than the grid method, as shown

by the comparison between these two methods in sec.4.3.2. As the extrema search is carried out in a restricted region R, this method can be called as the *restricted region method* .

However, no analytic means exist for finding this region R, except in the case of a class of uncertain systems known as *linear affine uncertain systems*. The restricted region method, as applied to this class is described in the following section.

4.3 Restricted region method as applied to the class of linear affine uncertain systems

In this section we will be needing the following definitions

Definition 4.3.1- A function $f_a: \mathbb{R}^n \longrightarrow \mathbb{R}$, is said to be linearly affine if it can be expressed as a sum of a linear function $g: \mathbb{R}^n \longrightarrow \mathbb{R}$ and a constant $k \in \mathbb{R}$, i.e.

$$f_a(\mathbf{q}) = g(\mathbf{q}) + k \ , \qquad \mathbf{q} \in \mathbb{R}^n.$$

Definition 4.3.2- Any system with a structured uncertainty model P(s) given in equation 2.2, chapter 2,is said to be a linear affine uncertain system, if each model coefficient a_i and b_i, are linearly affine in $\mathbf{q}$, the uncertain parameter vector.

Definition 4.3.3- Given the n - dimensional rectangle

$$Q = \{ \mathbf{q} \in \mathbb{R}^n \mid q_i^- \le q_i \le q_i^+ \ ; \ 1 \le i \le n\}$$

an edge of Q is defined as the set of vectors $E_i \le Q$, given as

$$E_i = \{ e \in Q \mid e_i^- \le e_i \le e_i^+ \text{ and } e_j = e_j^- \text{ or } e_j^+ ,\ j \ne i \} .$$

Now, as G(s) and F(s) are rational transfer functions, it is very easy to prove the following lemma -

Lemma 4.3.1 - [BARM93] If the uncertain system P(s) shown in figure 2.1 is a linear affine uncertain system, then all the system transfer functions for this feedback configuration (e.g. $T_{yr}(s)$, the complementary sensitivity function, etc.) are also linear affine uncertain systems.

Consequently, henceforth, we will concern ourselves with some general linear affine uncertain system, viz. V(s) with the knowledge that whatever results hold for V(s) will also hold for all the system transfer functions (including the open loop gain) .

Now use will be made of the following very important theorem proved in [FU90].

Theorem 4.3.1 - For a linear affine uncertain system V(s), at each trial frequency ω_o, the $V(j\omega_o,\mathbf{q})$ template boundary is the image of all or some edges E_i of Q.

The implication of this theorem is that, for linear affine uncertain systems, the frequency response extrema at each trial frequency lie on the edges of Q. The set of edges of Q, will then constitute the restricted region R, mentioned in the earlier section.

Extrema search along each edge E_i of Q, is essentially a one dimensional search as is evident from definition 4.3.3. There are very many one dimensional search algorithms [RAO84], of

which we chose the quasi Newton optimisation technique. Greater speed and accuracy, and the fact that it is the easiest to implement as far as NAG optimisation routines [NAG90] are concerned, were the reasons for choosing this particular optimisation technique. More specifically, a mini-max algorithm using the NAG routine E04JAF, an easy to use quasi Newton algorithm [NAG90], was developed in this thesis for handling linear affine uncertain systems. The extrema search is restricted to the edges of Q.

4.3.1 A Digression

A fairly recent paper by A.C.Bartlett [BART93],however, has shown that an extrema search of the kind just described above, for linear affine uncertain systems, is really not needed, as far as magnitude extremisation is concerned. Here a very simple method for finding the magnitude extrema using at most four values of $V(j\omega_o,\mathbf{q})$ along each edge E_i of Q, is given. We, however, persisted with the use of the NAG based extrema search algorithm for the following reason -

We observed that our algor thm,on an average ,made about 20 $f(\mathbf{q})$ value computations as against 5 (worst case) made by Bartlett s method. In other words, our algorithm, on an average, executed 5 times more the worst case number of arithmetic operations as executed in Bartlett s method. Now, even in a PC an increase by a factor of this magnitude in the number of arithmetic operations would be insignificant, computation time wise. So, efficiency wise, our algorithm can be said to

more or less as efficient as Bartlett s method.

4.3.2 Performance comparison of the restricted region method and the grid method

The test system chosen for this comparison was the one handled in example 5.3, chapter 5, viz.

$$P(s) = \frac{(s + a)}{s^3 + (13 + b)s^2 + (30 + 13b)s + 30b}$$

where $a \in [0.5, 2.5]$ and $b \in [4.0, 8.0]$.

For the grid method each uncertain parameter s interval was uniformly partitioned into 99 sub-intervals.

Frequency response computation for the system sensitivity

$$S(s) = \frac{1.0}{1.0 + L(s)}$$

over a set of 100 frequencies belonging to the interval [0.01,100] rad/sec. resulted in the following observations -

	Grid method	Restricted Region method
Time taken for each frequency (on an average)	8 minutes	12 seconds

Table 4.1. Comparison of the performance of the grid and the restricted region methods.

As expected the restricted region method was found to be quite fast relative to the grid method. These computations were done on a HP 9000 (series 800) super

mini computer.

4.3.3 Beyond the linear affine uncertainty case

Theorem 4.3.1 is the most advanced result concerning frequency response computation for systems with structured uncertainty [BART93]. For uncertain systems with functional dependence of the coefficients more complex and nonlinear than the linear affine case, to date, no analytical result exists which might ease the computational aspects. It is interesting to note the existence of a similar situation in the area of Kharitonov - like analysis of parametric uncertainty systems [BARM93].

In view of the existing scenario, for uncertain systems with uncertainty more complex than that encountered in the linear affine case, the grid method (section 4.2) seems to be the only alternative.

4.4 Nature of structured uncertainty handled by the software developed in this thesis

We will need the following definition in this section.

Definition 4.4.1 - An uncertain system with a structured uncertainty model given in equation 2.2, is said to be polynomially uncertain if each coefficient (a_i or b_i) of the model in equation 2.2 is some multivariable polynomial in **q**, the uncertain parameter vector, with real

coefficients.

Note that a linear affine uncertain system is a special case of polynomially uncertain systems.

The software developed in this thesis is equipped to handle arbitrary polynomially uncertain systems. The motivation for adoption of capability to handle this type of uncertain systems stems mainly from the following reasons -

(1) Most of the systems encountered e.g electrical, mechanical, etc., have models with coefficients which are some multivariable polynomials of the system parameters [THA60].

(2) For each coefficient, say a_i, such that

$$a_i = f_n(q) \quad , \; q \in Q$$

where $f_n(\;)$ is some non - polynomial, nonlinear continuous function of q, it is always possible, by the Weierstrass approximation theorem [SIM61], to find a polynomial g(q) with real coefficients such that

$$| \; f(q) - g(q) \; | \;\; \leq \;\; \varepsilon \; , \text{ over } Q$$

where $\varepsilon > 0$, is an arbitrarily small number. If ε is sufficiently small then we can take

$$a_i \; = \; g(q) \; , \text{ over } Q$$

and carry out the design for the resulting "modified" system. If the approximation effected is quite accurate then this design should hold for the actual system as well.

It may be noted that variable time delay systems can also be handled by the software developed, by going for rational Padé approximations for the " e^{-Ts} " terms involved.

The solutions to five examples solved using the algorithm described in this chapter (section 4.1) are presented in the next chapter.

CHAPTER 5

EXAMPLES

In this chapter we will discuss the solutions to five design problems, obtained using the mini-max approach to QFT described in the last chapter (see section 4.1, chapter 4).

Here, we will be needing the following definition -

Definition 5.1 - A frequency interval $W = [\omega_1, \omega_2]$ is said to *logspaced* into n sub-intervals if it is partitioned into n sub-intervals each of length $[\log_{10}(\omega_1/\omega_2)]/n$.

NOTE : This feature was adopted from a MATLAB utility called "logspace" [MAT87].

5.1 Example 1

Problem statement - A DC motor with an inertial load Jl is modelled by the transfer function [BAI91]

$$P(s) = \frac{\omega_L(s)}{e(s)} = \frac{0.2}{(a_2)s^2 + (a_1)s + 0.04}$$

where "ω_L" is the motor shaft speed and "e" is the motor armature voltage and where

$$a_1 = 0.056 \times 10^{-2} + 0.4(Jl)$$

$$a_2 = 3 \times 10^{-6} + 22 \times 10^{-4}(Jl)$$

The load moment of inertia Jl is assumed to vary in the interval $[0.7\times10^{-4}; 14\times10^{-3}]$ kg-m^2.

The performance goal of interest here is to have the motor shaft speed track a speed reference signal "r". Response speed and damping considerations lead to the

following time domain bounds to be specified -

SP1 - 2% settling time of the unit step response $t_s \in [6.0, 8.0]$ seconds.

SP2 - Peak overshoot of the unit step response $M_p \leq 20\%$.

NOTE : The assumption of a lower bound on t_s allows us to set up the problem, relatively easily, as one in which the frequency domain specifications are of the form in equation 2.5(a). Even though this specification may seem unrealistic, it should be noted that our main objective here was to illustrate the use of this software rather than solve a real world design problem.

Solution - The design procedure consisted of the following steps -

STEP 1 Setting up of the frequency domain specifications - The time to frequency domain conversion needed for this problem is described in example 2.1, chapter 2. The resulting bounding transfer functions, as found there, are

$$T^u_{yr}(s) = \frac{(0.216s + 1.08)}{s^2 + 0.94s + 1.08}$$

$$T^l_{yr}(s) = \frac{1.35}{s^3 + 1.998s + 2.736s + 1.35}$$

The equivalent performance specifications in the frequency domain are now to design a G(s) and a F(s) to ensure

$$|T^l_{yr}(j\omega)| \leq |T_{yr}(j\omega)| \leq |T^u_{yr}(j\omega)|, \quad \forall\omega \geq 0 \qquad (5.1)$$

(see section 2.2, chapter 2).

STEP 2 Choice of the trial frequencies set Ω and an initial guess for G(s) -

Ω was chosen to be a set of 100 logspaced frequencies (see definition 5.1) in the (arbitrarily chosen) range of

[0.1,1000] rad/sec.

$G(s) = 1.0$ was the initial guess for G(s).

STEP 3 Synthesis of G(s) -

The initial guess of $G(s) = 1.0$ was found to be satisfactory in that it ensured

$$\Delta Lm[T_{yr}(j\omega)]_{max} \leq Lm[T^{u}_{yr}(j\omega)] - Lm[T^{l}_{yr}(j\omega)], \quad \forall\omega \in \Omega \qquad (5.2)$$

It also resulted in a stable closed loop set-up. This particular fact can be verified using the Routh-Hurwitz test.

Thus simple closing of the loop around P(s) was found to be satisfactory, performance and stability wise.

STEP 4 Synthesis of the pre-filter F(s) -

Bounds on the magnitude of the pre-filter F(s) were computed at all the trial frequencies, using equation 3.9, chapter 3. Use of these bounds and MATLAB lead to a design of

$$F(s) = \frac{1.2}{s^2 + 1.3s + 1.0}$$, for the pre-filter.

Time domain testing of this design using MATLAB validated it as can be seen from the unit step responses ,in figure 5.1, obtained for a set of 10 values of the uncertain parameter J1 (also see table 5.1).

The satisfaction of equation 5.1, for all values taken by J1 is shown in figure 5.2.

Stability can be established via the fuzzy open loop polar plot, shown by means of a finite set of discrete ABCD(ω_0) - type templates (see section 4.1, chapter 4) in figure 5.3.

Parameter value	Settling time (sec)	Peak overshoot (%)
0.7e-4	6.03	7
1.4e-4	6.03	7
2.8e-4	6.03	7
35e-4	6.03	7
54e-4	6.03	7
77e-4	6.03	7
91e-4	6.03	7
10e-3	6.03	7
12e-3	6.03	7
14e-3	6.06	7

Table 5.1 Settling time and peak overshoot of the unit step response for different values of the uncertain parameter J1 - Example 1.

5.2 Example 2

Problem Statement - The output of an unstable uncertain system

$$P(s) = \frac{1.0}{(s - a)(s + 10)}, \quad a \in [1.0, 5.0]$$

has to satisfy tracking specifications same as those given in example 5.1, viz. SP1 and SP2.

Solution - The design procedure consisted of the following steps -

STEP 1 Setting up of the frequency domain specifications - Since the time domain specifications are the same as in example 5.1, the equivalent frequency domain specification

is, as in example 5.1, to design a $G(s)$ and $F(s)$ so as to ensure that $T_{yr}(s)$ satisfies equation 5.1 with the bounding transfer functions $T^{u}_{yr}(s)$ and $T^{l}_{yr}(s)$ being the same as in example 5.1.

STEP 2 Choice of the trial frequencies set Ω and an initial guess for $G(s)$ -

Ω was chosen to be a set of 100 logspaced frequencies in the (arbitrarily chosen) range [0.1,1000] rad/sec. .

$G(s) = 1.0$ was the initial guess for $G(s)$.

STEP 3 Synthesis of $G(s)$ -

The initial guess of $G(s) = 1.0$ indicated that the maximum lower bound on $|G(j\omega)|$, $\forall\omega \in \Omega$, was 3565 .

A candidate $G(s) = 3565.0$ was consequently found to be satisfactory performance wise (i.e. it satisfied equation 5.2) and also ensured a stable closed loop system. In order to, however, provide a "realistic" design involving a strictly proper $G(s)$, we added a simple pole to this candidate $G(s)$ giving

$$G(s) = \frac{3565(b)}{s + b}, \qquad b > 0.$$

However to ensure stability we had to take a very large value for b (greater than 1000). In order to keep b not too large and at the same time ensure stability, we additionally resorted to a lead compensator resulting, for b = 300, in the following design for $G(s)$

$$G(s) = \frac{34.224 \times 10^{5}(s + 25)}{(s + 80)(s + 300)}$$

This $G(s)$ was found to satisfy equation 5.2 as well as ensured closed loop stability.

STEP 4 Synthesis of the pre-filter $F(s)$ -

Bounds on the magnitude of the pre-filter F(s) were found using equation 3.9, chapter 3. MATLAB was then used to obtain the following design for the pre-filter

$$F(s) = \frac{1.25}{s^2 + 1.35s + 1.25}$$

Time domain testing of this design using MATLAB validated it, as can be seen from the unit step responses in figure 5.4, obtained for a set of ten values of the uncertain parameter "a" (also see table 5.2).

The satisfaction, by this design, of the frequency domain specification of equation 5.1 is shown in figure 5.5.

Stability can be established from the fuzzy open loop polar plot, consisting of a finite number of discrete ABCD(ω_o) - type templates and shown in figures 5.6 and 5.7.

Parameter values	Settling time (sec.)	Peak overshoot(%)
1.0	5.34	9.7
5.0	5.34	9.44
1.5	5.34	9.54
4.5	5.34	9.6
2.1	5.34	9.35
4.1	5.34	9.72
3.2	5.34	9.01
3.5	5.34	8.92
1.25	5.34	9.61
4.7	5.34	9.54

Table 5.2 Settling time and peak overshoot of the unit step response for different values of the uncertain parameter a -Example 2.

5.3 Example 3

Problem Statement - Given the system

$$P(s) = \frac{(s + a)}{(s + 3)(s + 10)(s + b)},$$

$$a \in [0.5, 2.5], \quad b \in [4.0, 8.0]$$

find a stabilising compensator G(s) such that the sensitivity function $S(s) = \dfrac{1.0}{1.0 + L(s)}$ satisfies

$$|S(j\omega)| \leq 0.01, \quad \forall \omega \in [0, 0.3] \text{ rad/sec.}$$

Solution - The design procedure consisted of the following steps -

STEP 1 Setting up of the frequency domain specifications -

The design objective was to design a G(s) to ensure

$|S(j\omega)| \leq 0.01, \quad \forall\omega \in [0, 0.3]$ - given

and $|S(j\omega)| \leq 100, \quad \forall\omega > 0.3$ - this specification was arbitrarily made keeping in mind that

$$|S(j\omega)|_{max} > 1.0 \qquad (5.3)$$

STEP 2 Choice of the trial frequencies set Ω and an initial guess for G(s) -

Ω was chosen to consist of a set of 100 logspaced frequencies in the (arbitrarily chosen) range of [0.01,100] rad/sec. .

G(s) = 1.0 was an initial guess for G(s).

STEP 3 Synthesis of G(s) -

The initial guess of G(s) = 1.0 indicated that the maximum lower bound on $|G(j\omega)|$ for all the trial frequencies was 4320 . Consequently we chose a candidate G(s)

$$G(s) = \frac{4320(b)}{(s + b)} \quad , \quad b > 0$$

"Tuning" the value of b we finally settled for b = 350.0 (arbitrary choice) resulting in

$$G(s) = \frac{15.12 \times 10^5}{(s + 350)}$$

which satisfied the performance specification of equation 5.3 as well ensured a stable closed loop system. This can be clearly seen from figures 5.8 - 5.11.

5.4 Example 4

Problem Statement - For the given system

$$P(s) = \frac{a}{(s + 2)(s + b)} \quad , \quad a,b \in [1.0,10.0]$$

beset with impulse-like disturbances (like e.g. voltage spikes) at the plant input, a compensator G(s) is sought such that the system response to these disturbances has a 2% settling time of less than a second, for all values of the uncertain parameters a and b.

Solution - The design procedure consisted of the following steps -

<u>STEP 1</u> Setting up of the frequency domain specifications -

We assumed unit impulse disturbances at the plant input. Consequently, the design objective was to ensure that the impulse response of the transfer function

$$T_{yd1}(s) = \frac{y(s)}{d1(s)} = \frac{P(s)}{1.0 + L(s)} \qquad (5.4)$$

(see figure 2.1, chapter 2) has a 2% settling time of less than a second for all values of the uncertain parameters a and b.

To have such a settling time, as a rule of thumb, we

define a bounding transfer function $T^{u}_{yd1}(s)$ with a settling time of less than 1 second, such that ensuring

$$|T_{yd1}(j\omega)|_{max} \le |T^{u}_{yd1}(j\omega)|, \quad \forall\omega \ge 0 \qquad (5.5)$$

would ensure the disturbance response settling time specification. This frequency domain specification of equation 5.5 was made on the same lines as suggested in [D'AZ88] and [JAY92].

Consequently we considered a boundary model of the form

$$T^{u}_{yd1}(s) = \frac{\omega_n^2}{s^2 + (2\xi\omega_n)s + \omega_n^2} \qquad (5.6)$$

It can be easily verified that the 2% settling time for this model is given as

$$t_s = \frac{\log_e(50\omega_n)}{\xi\omega_n}$$

We arbitrarily assumed $\omega_n = 14$ rad/sec (which is far greater than the average system bandwidth of about less than 2 rad/sec.). This fixed up $\xi \ge 0.47$ for $t_s \le 1$ second. As a result, we assumed $\xi = 0.5$. Thus the boundary model used was

$$T^{u}_{yd1}(s) = \frac{196}{s^2 + 14s + 196},$$

with a settling time of 0.94 sec. .

<u>STEP 2</u> Choice of the trial frequencies set Ω and an initial guess for G(s) -

Ω was chosen to be a set of 100 logspaced frequencies in the (arbitrarily chosen) range [0.01,100] rad/sec.

G(s) = 1.0 was an initial choice for G(s).

<u>STEP 3</u> Synthesis of G(s) -

The initial guess of G(s) = 1.0 indicated that a gain

of more than 1.08 was required over a certain low frequency range, essentially at around 3 rad/sec. Based on this a controller candidate

$$G(s) = \frac{45.0}{(s + 30)}$$

was considered which was found to to satisfy equation 5.5 as well as ensured a stable closed loop system.

Time domain testing using MATLAB, however, showed that for low values of the parameters a and b (e.g. a = b = 1.0) the resulting settling time was, contrary to expectations, greater than 1 second.

Investigation using the root locus indicated that this could possibly be due to the relative closeness of the closed loop poles to the $j\omega$ - axis, for low values of the uncertain parameters a and b.

Now, note that the two poles of P(s) can be weaned away from the $j\omega$ - axis by getting G(s) to have two LHP zeros. These two zeros were chosen to be located at $- 8 \pm j3$. The idea behind this choice was that if the resulting loop function L(s) has a dominant pole pair at these locations then the settling time t_s would be less than 1 second. This emerges from the fact that a second order system with the same model as in equation 5.6 and with the poles located at these locations would have a settling time of less than a second (specifically about 0.76 sec).

To make this G(s) realisable we added two "far-off" poles at $- 41 \pm j10$, resulting in the following design

$$G(s) = \frac{292.8(s^2 + 16s + 73)}{(s^2 + 82s + 1781)}$$

The gain factor of 292.8 was required to "push" the open loop poles towards the zeros at $-8 \pm j3$.

This G(s) was found to ensure satisfaction of equation 5.5, as well as closed loop stability.

Time domain testing for a set of ten value pairs of a and b (see table 5.3) also validated this design. The resulting impulse responses are shown in figure 5.12.

Satisfaction of the frequency domain specification of equation 5.5 is shown in figure 5.13.

Figures 5.14 and 5.15 show the resulting fuzzy open loop polar plot consisting of a finite set of discrete ABCD(ω_0) - type templates which were used in establishing closed loop stability.

Parameter value - a	Parameter value - b	Settling time (sec)
1.0	1.0	0.95
10.0	10.0	0.61
2.0	9.0	0.71
9.0	2.0	0.56
3.5	6.5	0.68
6.5	3.5	0.6
4.7	7.3	0.65
7.3	4.7	0.6
5.5	8.2	0.64
8.2	5.5	0.6

Table 5.3 Settling time of the unit impulse response for different values of the uncertain parameters a and b - Example 4.

5.5 Example 5

Problem Statement - For the given system

$$P(s) = \frac{-s + a}{(s + 3)(s + 10)(s + b)}$$

$$a \in [0.5, 2.5] \text{ and } b \in [4.0, 8.0]$$

beset with impulse-like disturbances at the plant output, a compensator G(s) is sought such that the system response to these disturbances has a 2% settling time of less than a second for all values of the uncertain parameters a and b.

Solution - The design procedure consisted of the following steps -

STEP 1 Setting up of the frequency domain specifications -

We assumed unit impulse disturbances at the plant output. Consequently, the design objective was to ensure that the impulse response of the following transfer function

$$T_{yd2}(s) = \frac{y(s)}{d2(s)} = \frac{1.0}{1.0 + L(s)} \qquad (5.7)$$

(see figure 2.1, chapter 2) has a settling time of less than a second for all the values of the uncertain parameters a and b.

As in example 5.4, we needed a boundary transfer function $T^{u}_{yd2}(s)$ with a settling time of less than 1 second such that ensuring of the following frequency domain specification

$$\left| T_{yd2}(j\omega) \right|_{max} \leq \left| T^{u}_{yd2}(j\omega) \right| , \quad \forall\omega \geq 0 \qquad (5.8)$$

would, as a rule of thumb, ensure the disturbance response settling time specification.

We, on similar lines as in [D AZ88] chose the following boundary transfer function model

$$T^{u}_{ydz}(s) = \left[\frac{(s + \alpha)}{(s + \alpha)^2 + \beta^2}\right]\frac{\alpha^2 + \beta^2}{\alpha} \qquad (5.9)$$

with the 2% settling time given as

$$t_s = \frac{\log_e[50(\alpha^2 + \beta^2)/\alpha]}{\alpha}$$

For a settling time of less than 1 second we chose (arbitrarily) $\alpha = 10.0$ and $\beta = 50.0$, resulting in the following boundary transfer function

$$T^{u}_{ydz}(s) = \frac{260.0(s + 10)}{s^2 + 20s + 2600}$$

with a settling time of 0.95 sec.

STEP 2 Choice of the trial frequencies set Ω and an initial guess for G(s) -

Ω was chosen to be a set of 100 logspaced frequencies in the (arbitrarily chosen) range of [0.01,100] rad/sec.

G(s) = 1.0 was the initial choice for G(s).

STEP 3 Synthesis of G(s) -

The initial guess of G(s) = 1.0 was found to be satisfying equation 5.8 and also ensured a stable closed loop system. But MATLAB aided time domain testing of this design showed that for low values of a and b, the resulting settling time was,contrary to expectations, greater than 1 second.

Investigation using the root locus, as done in example 5.4, indicated that this could possibly be due to the relative closeness of some closed loop poles to the $j\omega$ - axis for low values of the uncertain parameters a and b.

Based on this investigation, another candidate G(s)

given as

$$G(s) = \frac{8.58(s + 3.5)}{(s + 30)}$$

was considered. The zero at -3.5 was added to "take care" of of the open loop pole at -3.0. Note that due to the non-minimumphase open loop zero at a, we have the complementary root locus [KUO91]. The pole at -30.0, of G(s) was simply a "far-off" pole added to make G(s) realisable. Observe that this G(s) is a lead compensator.

This G(s) was found to satisfy equation 5.8 and also ensure a stable closed loop system. This design was also validated by MATLAB aided time domain testing for a set of ten value pairs of the uncertain parameters a and b (see table 5.4). The resulting impulse responses are shown in figure 5.16.

The satisfaction of the frequency domain specification of equation 5.8 is shown in figure 5.17.

The resulting fuzzy open loop polar plot consisting of a finite set of discrete ABCD(ω_0) - type templates is shown in figure 5.18.

Parameter value - a	Parameter value - b	Settling time (sec)
0.5	4.0	0.55
2.5	8.0	0.52
0.5	8.0	0.47
2.5	4.0	0.77
1.5	6.0	0.58

Table 5.4 - continued on the next page

Parameter value - a	Parameter value - b	Settling time (sec)
1.0	7.0	0.52
2.0	5.0	0.66
1.8	4.5	0.69
2.2	6.2	0.59
0.8	7.8	0.48

Table 5.4 (contd.) Settling time of the unit impulse response for different values of the uncertain parameters a and b - Example 5.

Thus in this chapter we described the solutions to five examples, obtained using the mini-max approach based QFT design scheme described in the previous chapter.

Though the semi-heuristical conversion of time domain specifications to the frequency domain worked in the first two examples, it however ran into trouble in the last two examples. One reason for this could have been improper choice of the boundary transfer functions. This particular aspect, however, unfortunately could not be investigated further. But the last two examples certainly sound a cautionary note regarding the setting up of frequency domain specifications equivalent to a given set of time domain specifications.

FIGURE 5.1 STEP RESPONSE --- EXAMPLE 1

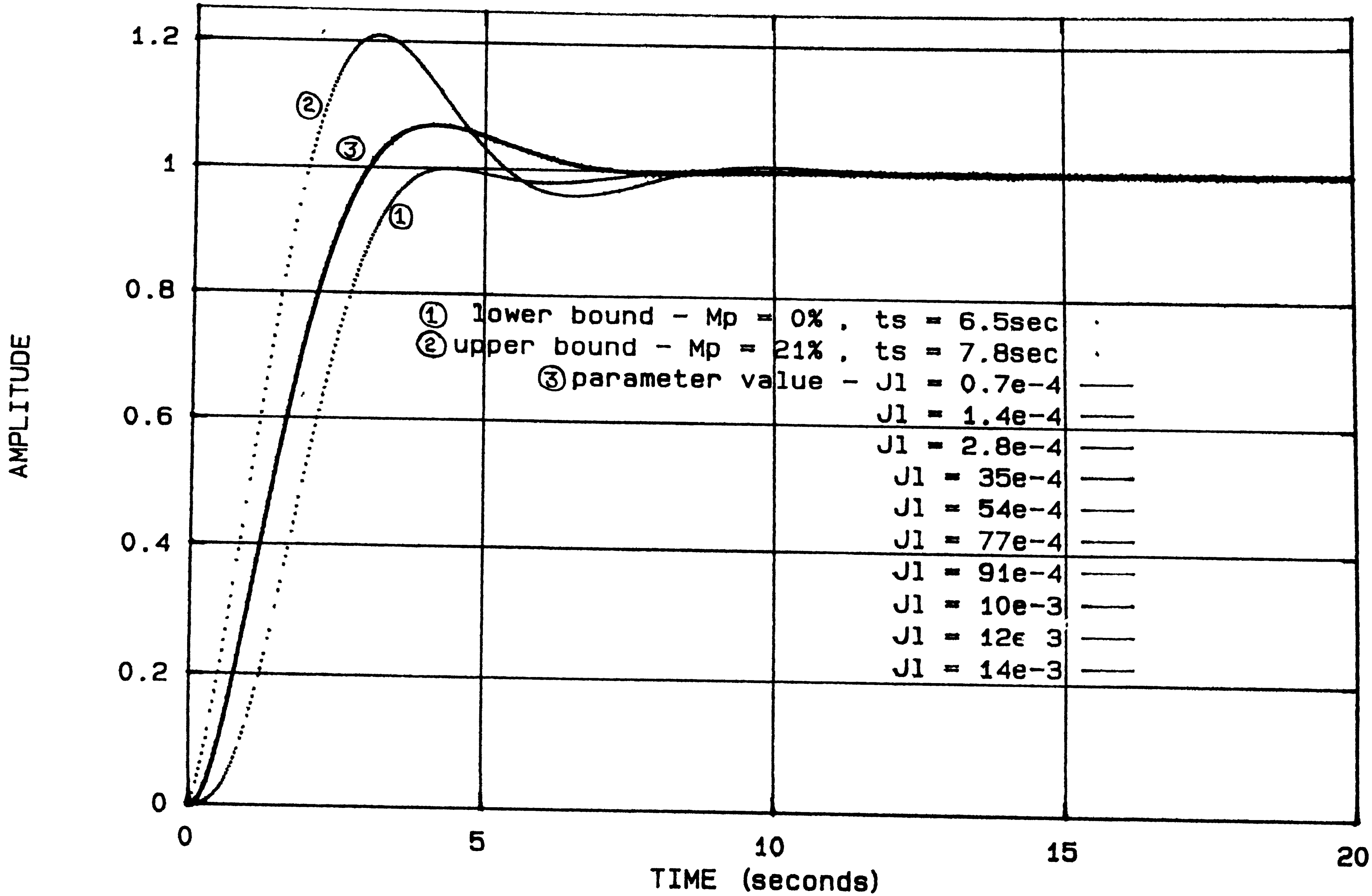
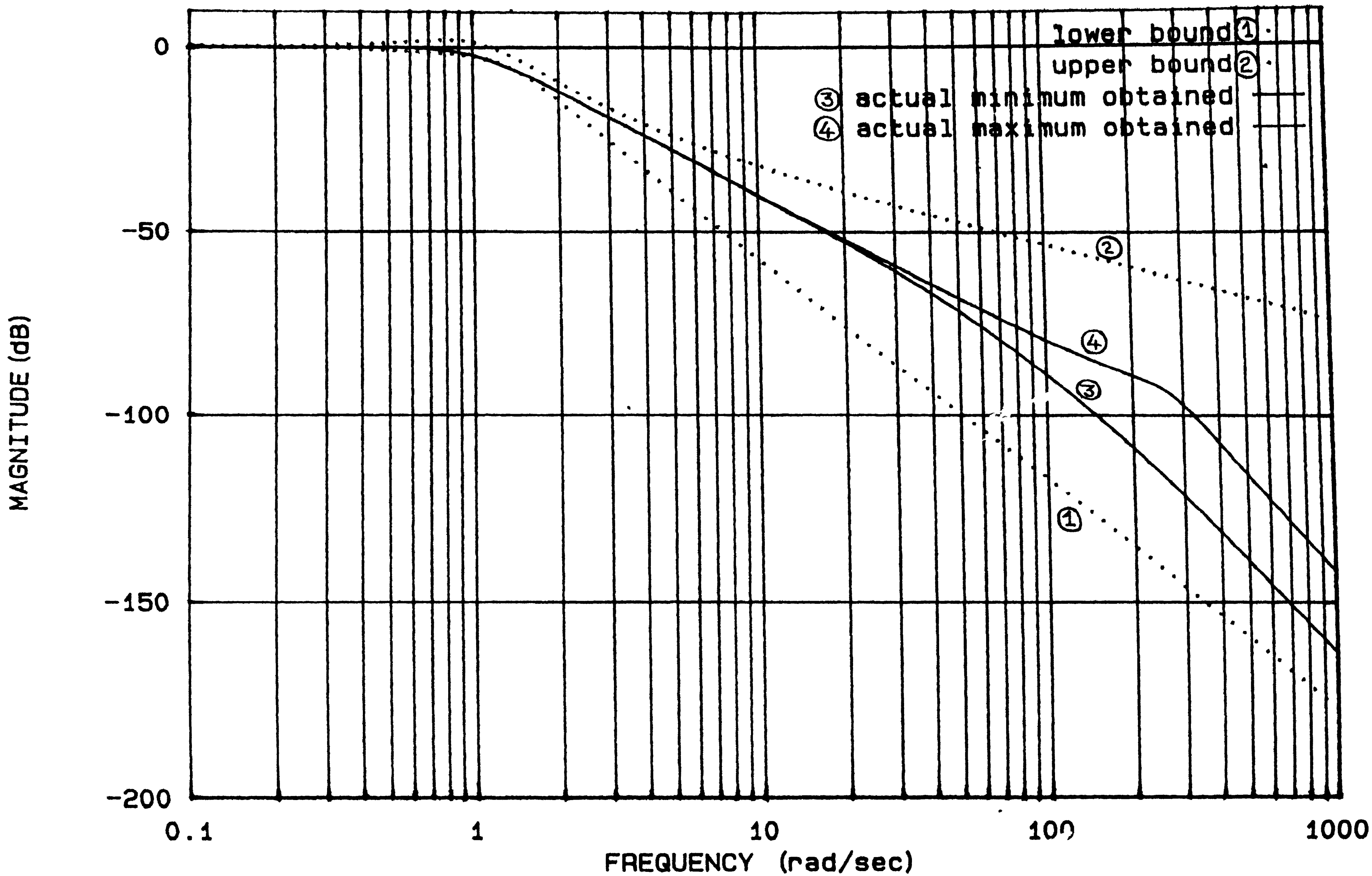


FIGURE 5.2 BODE MAGNITUDE PLOT OF ALLOWED AND OBTAINED CLOSED LOOP GAIN - EXAMPLE 1



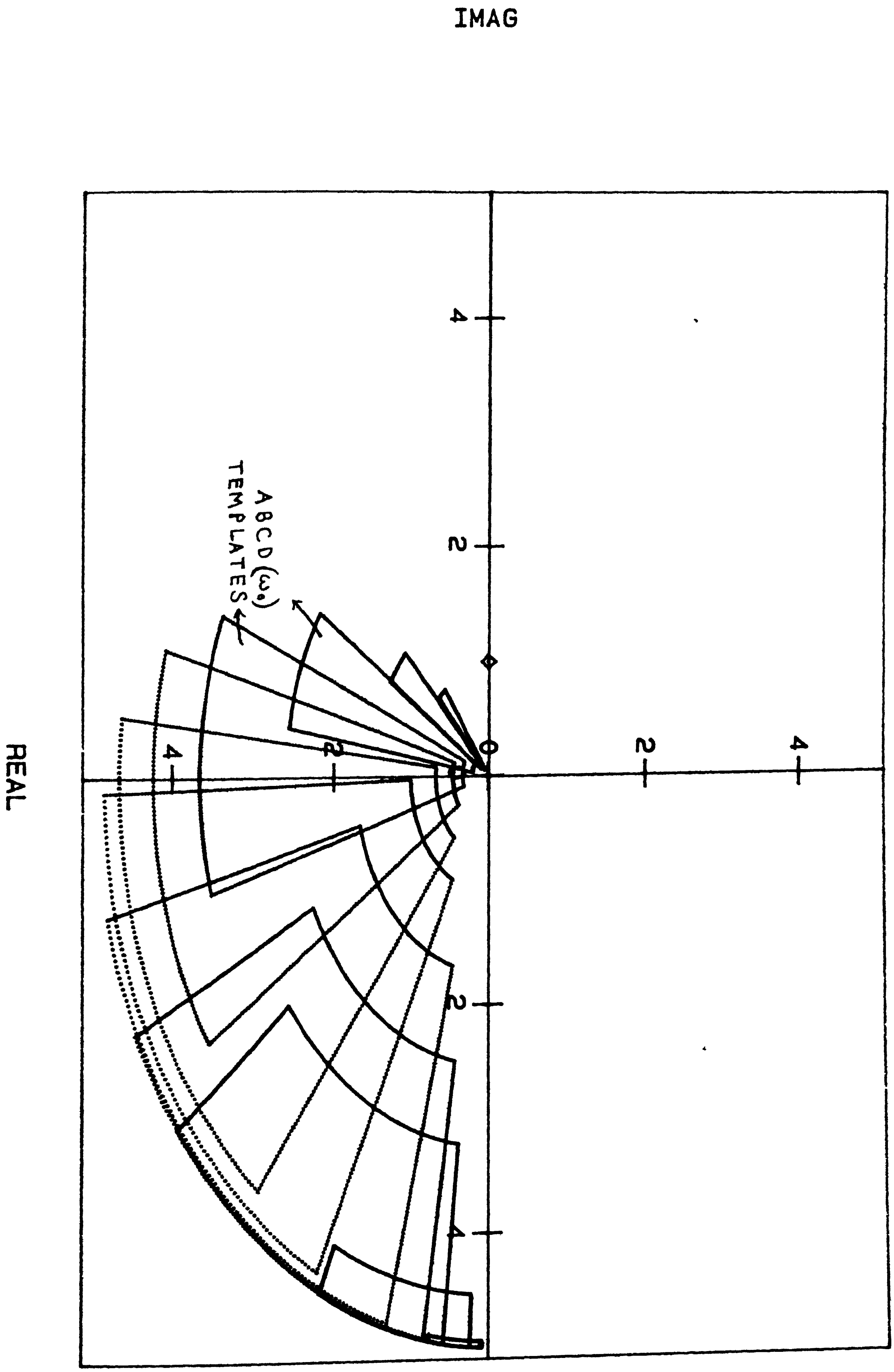


FIGURE 5.3 POLAR PLOT OF OPEN LOOP GAIN --- EXAMPLE 1

FIGURE 5.4 STEP RESPONSE --- EXAMPLE 2

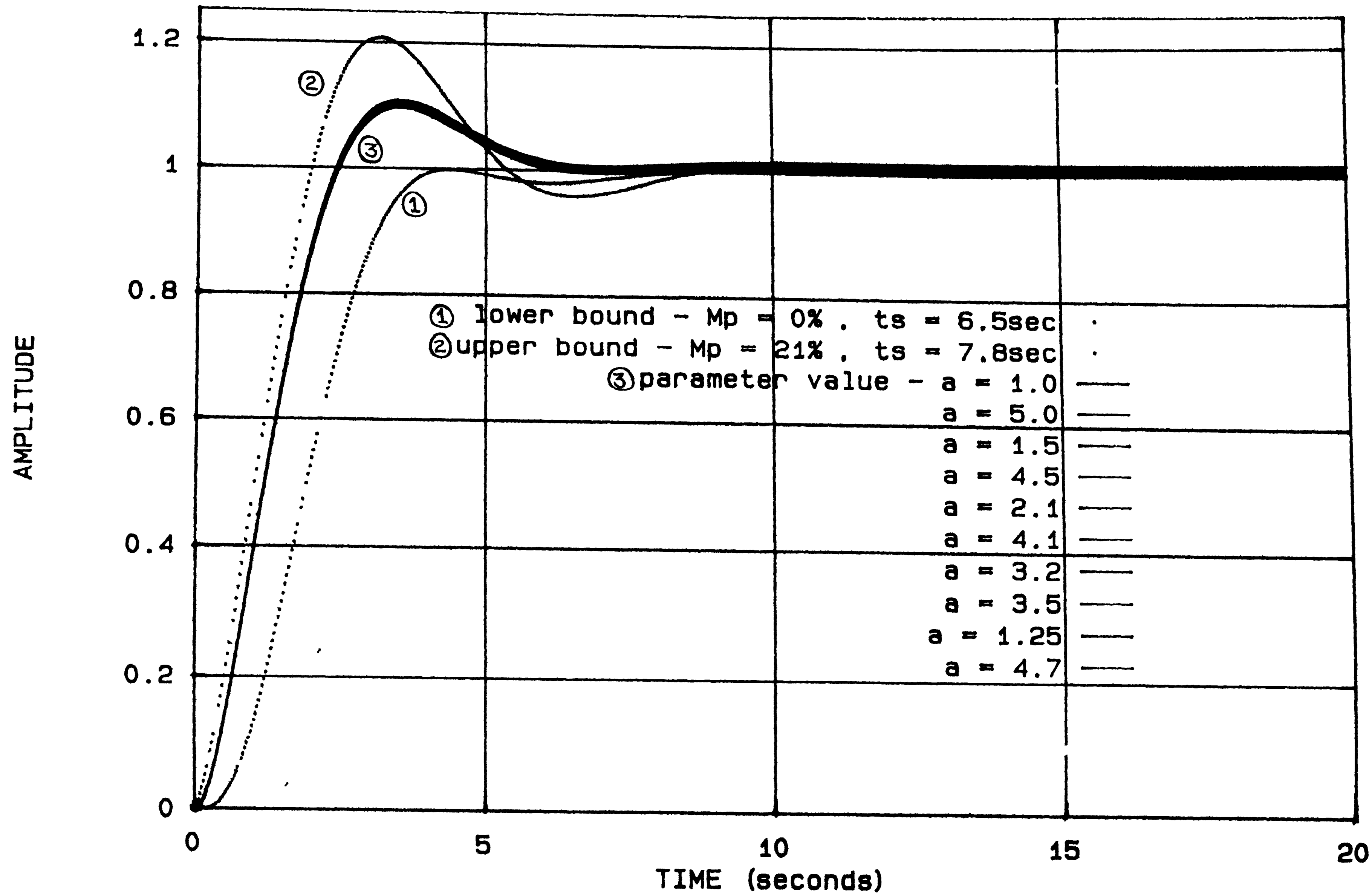


FIGURE 5.5 BODE MAGNITUDE PLOT OF ALLOWED AND OBTAINED CLOSED LOOP GAIN - EXAMPLE 2

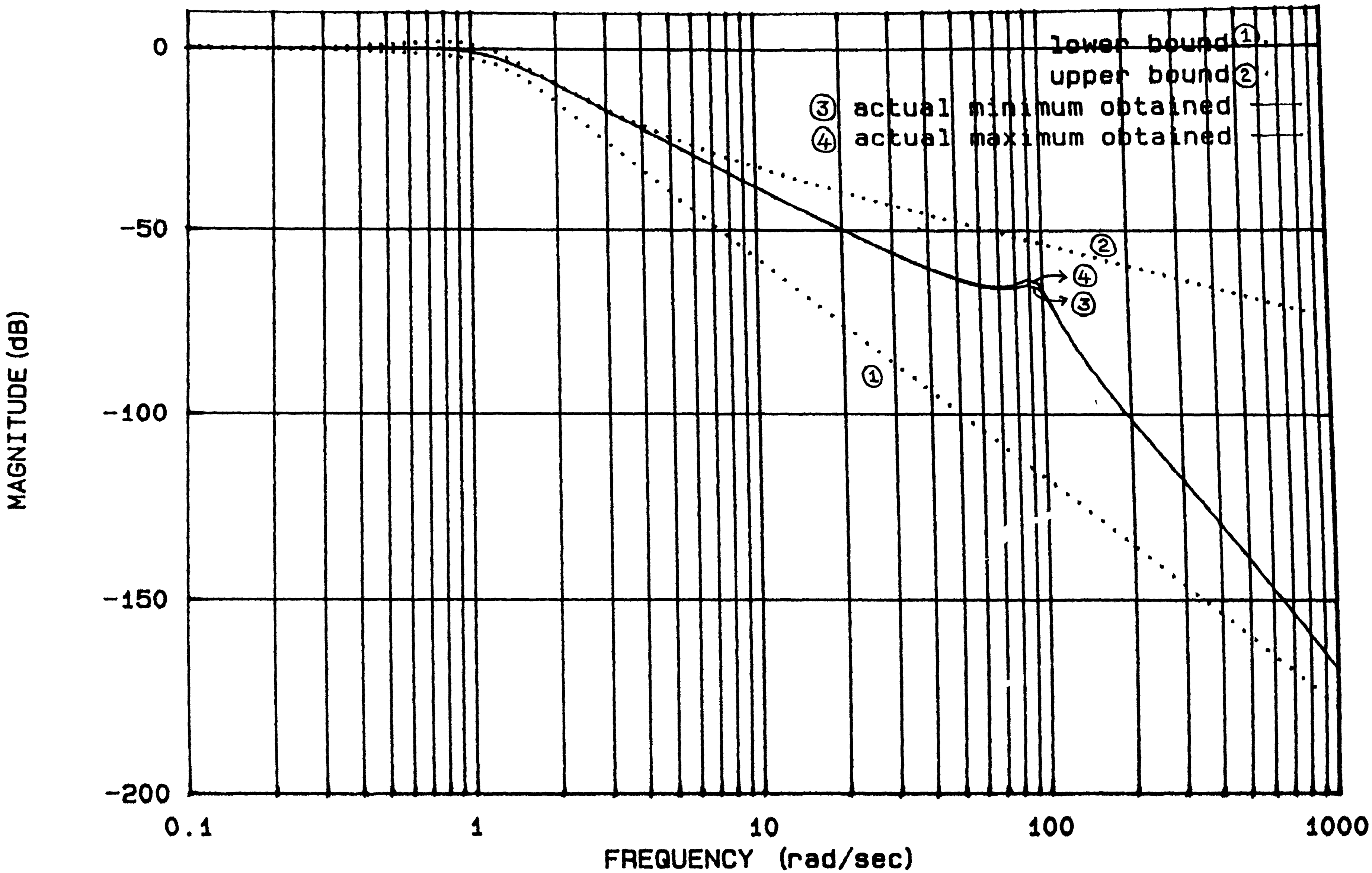
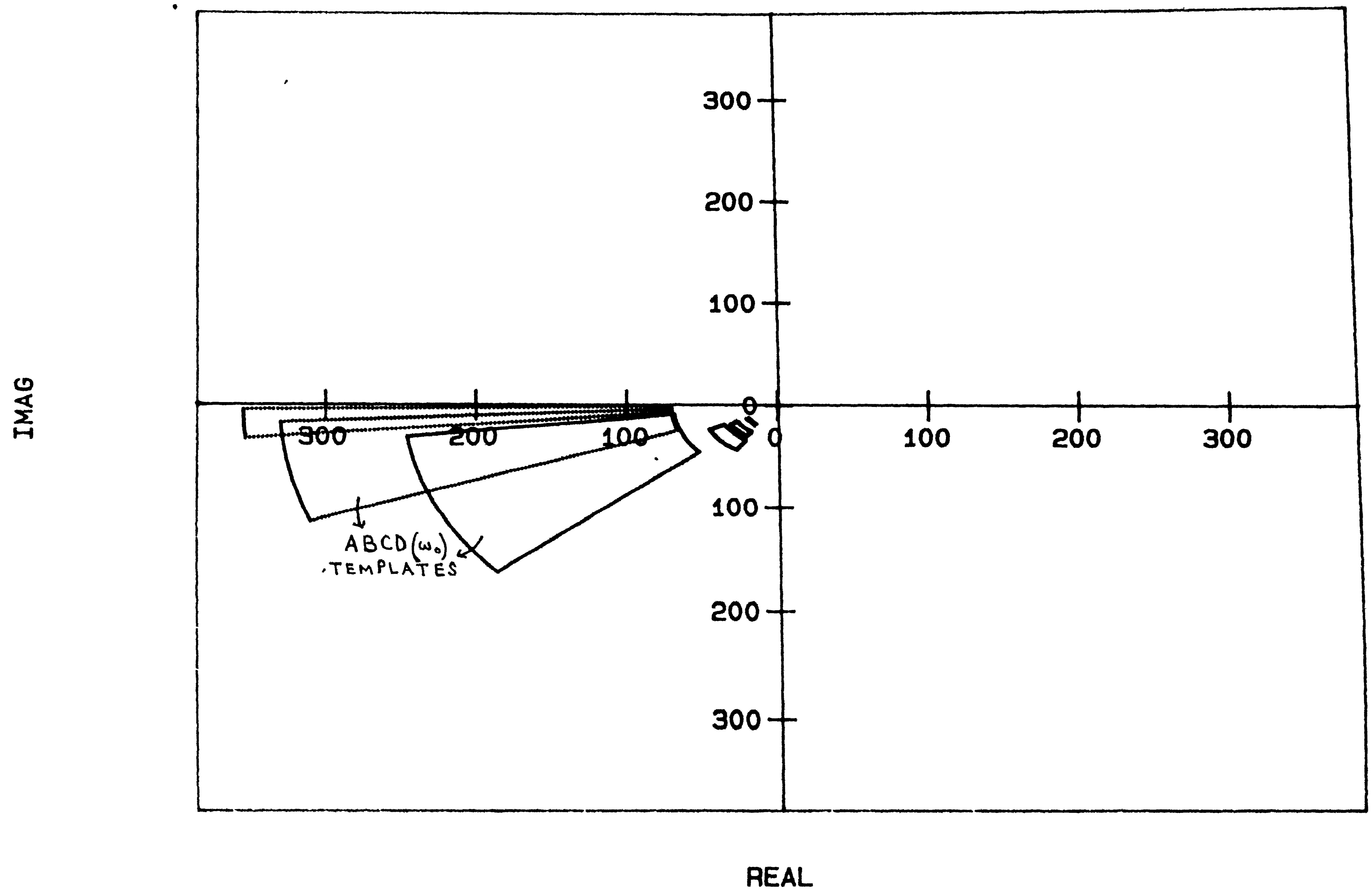


FIGURE 5.6 POLAR PLOT OF OPEN LOOP GAIN --- EXAMPLE 2



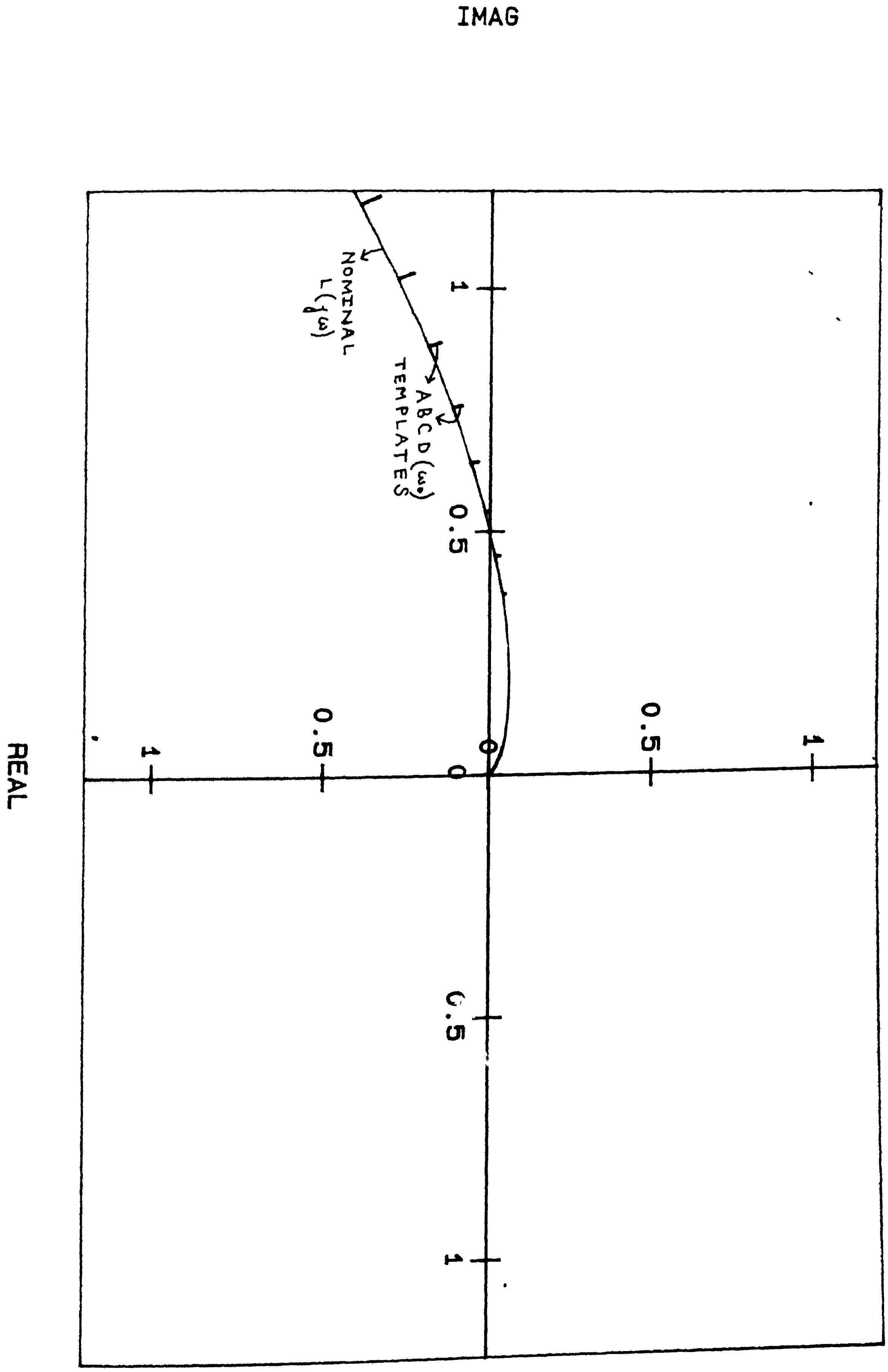


FIGURE 5.7 POLAR PLOT (MAGNIFIED) OF OPEN LOOP GAIN --- EXAMPLE 2

FIGURE 5.8 BODE MAGNITUDE PLOT OF ALLOWED AND OBTAINED SENSITIVITY - EXAMPLE 3

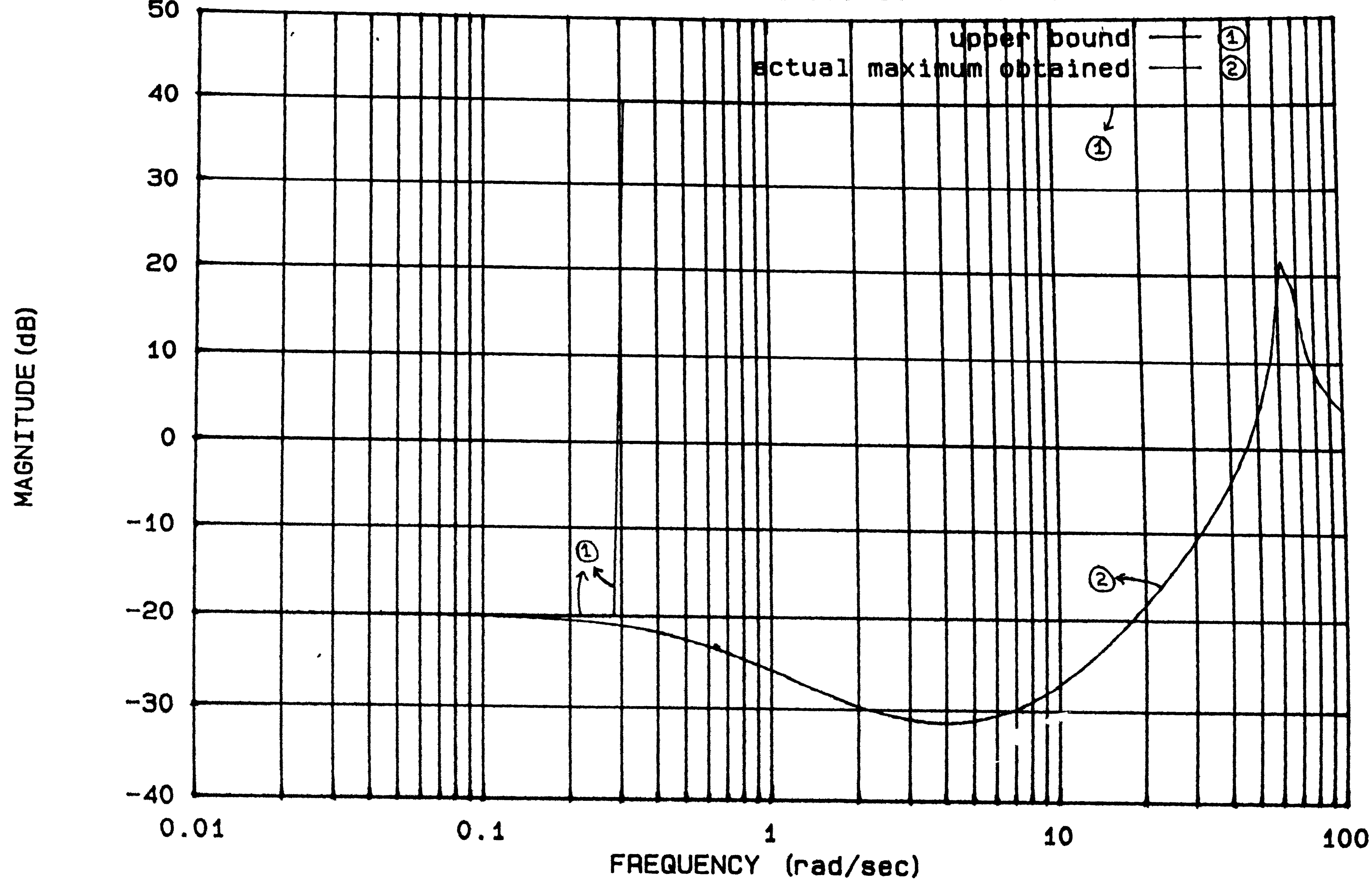


FIGURE 5.9 POLAR PLOT OF OPEN LOOP GAIN --- EXAMPLE 3

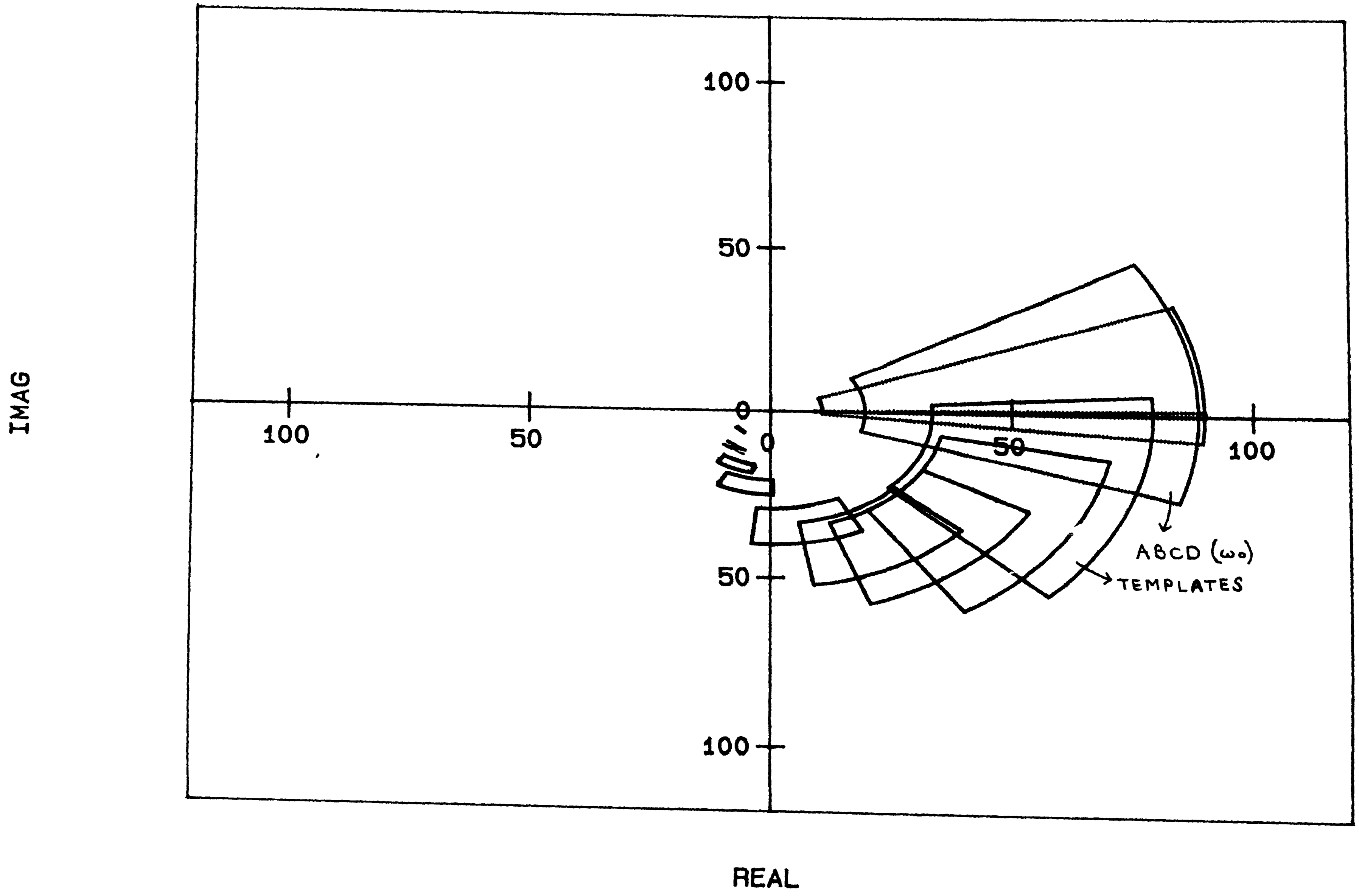


FIGURE 5.10 POLAR PLOT (MAGNIFIED) OF OPEN LOOP GAIN --- EXAMPLE 3

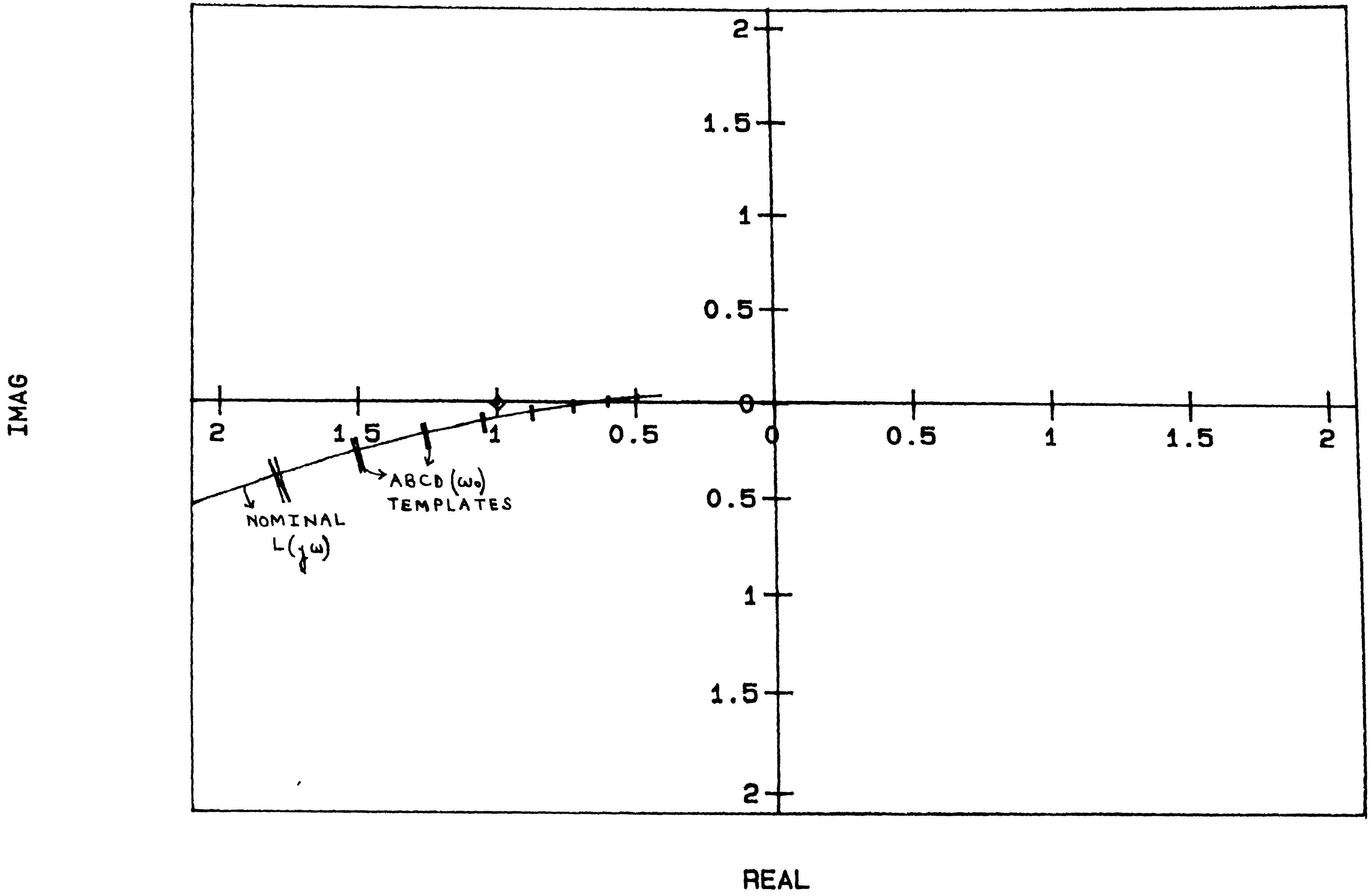
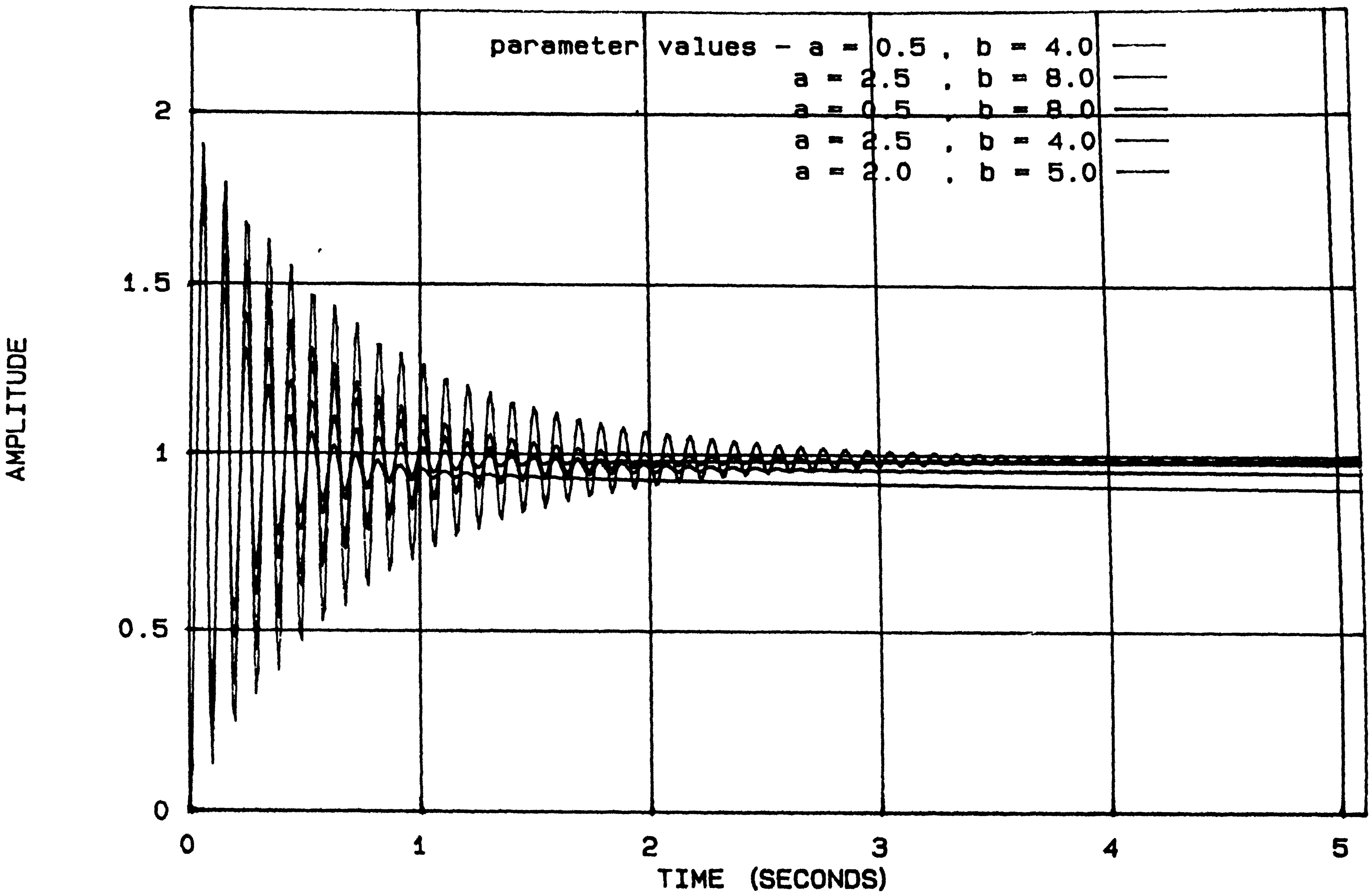


FIGURE 5.11 STEP RESPONSE --- EXAMPLE 3



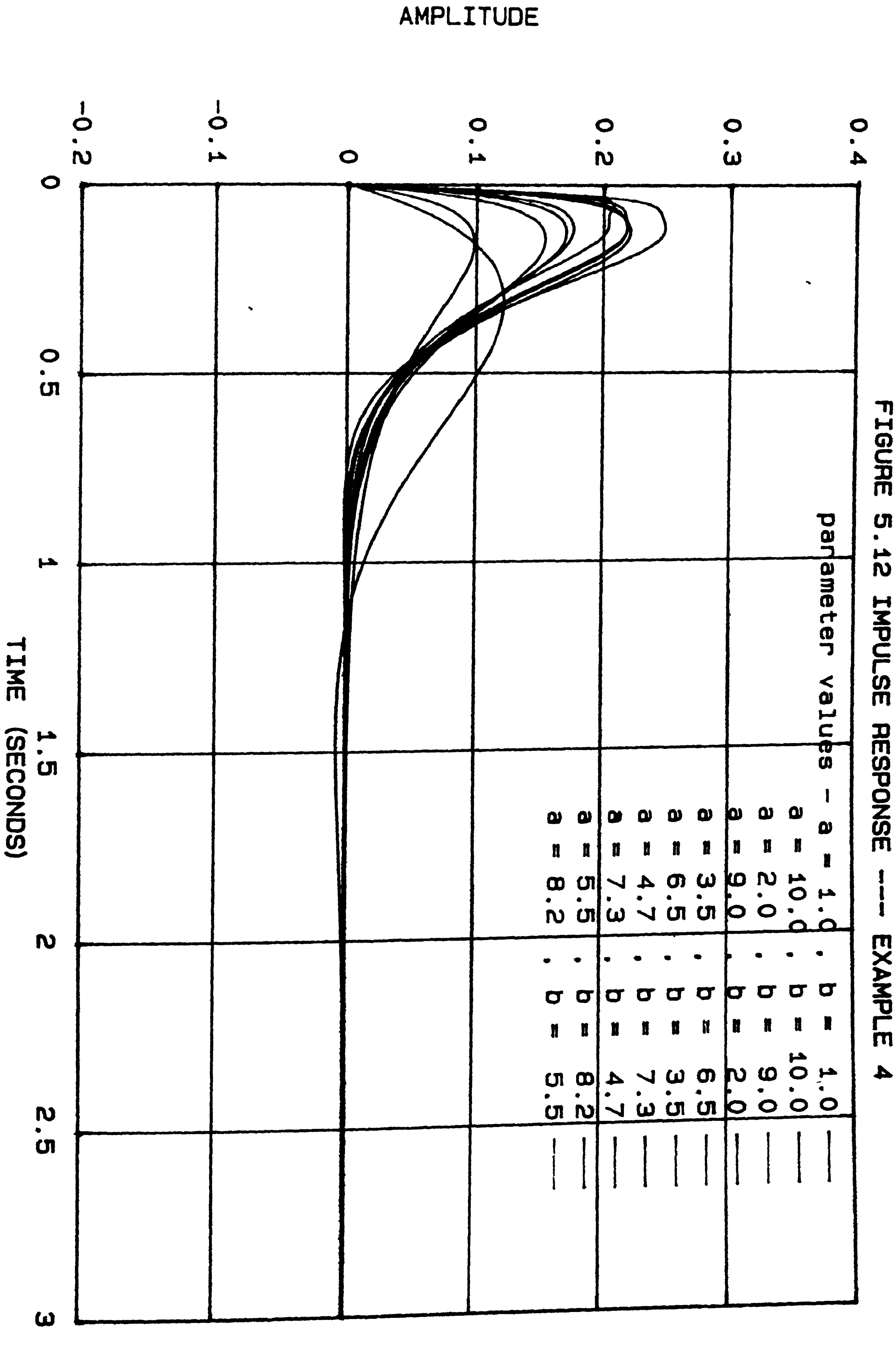
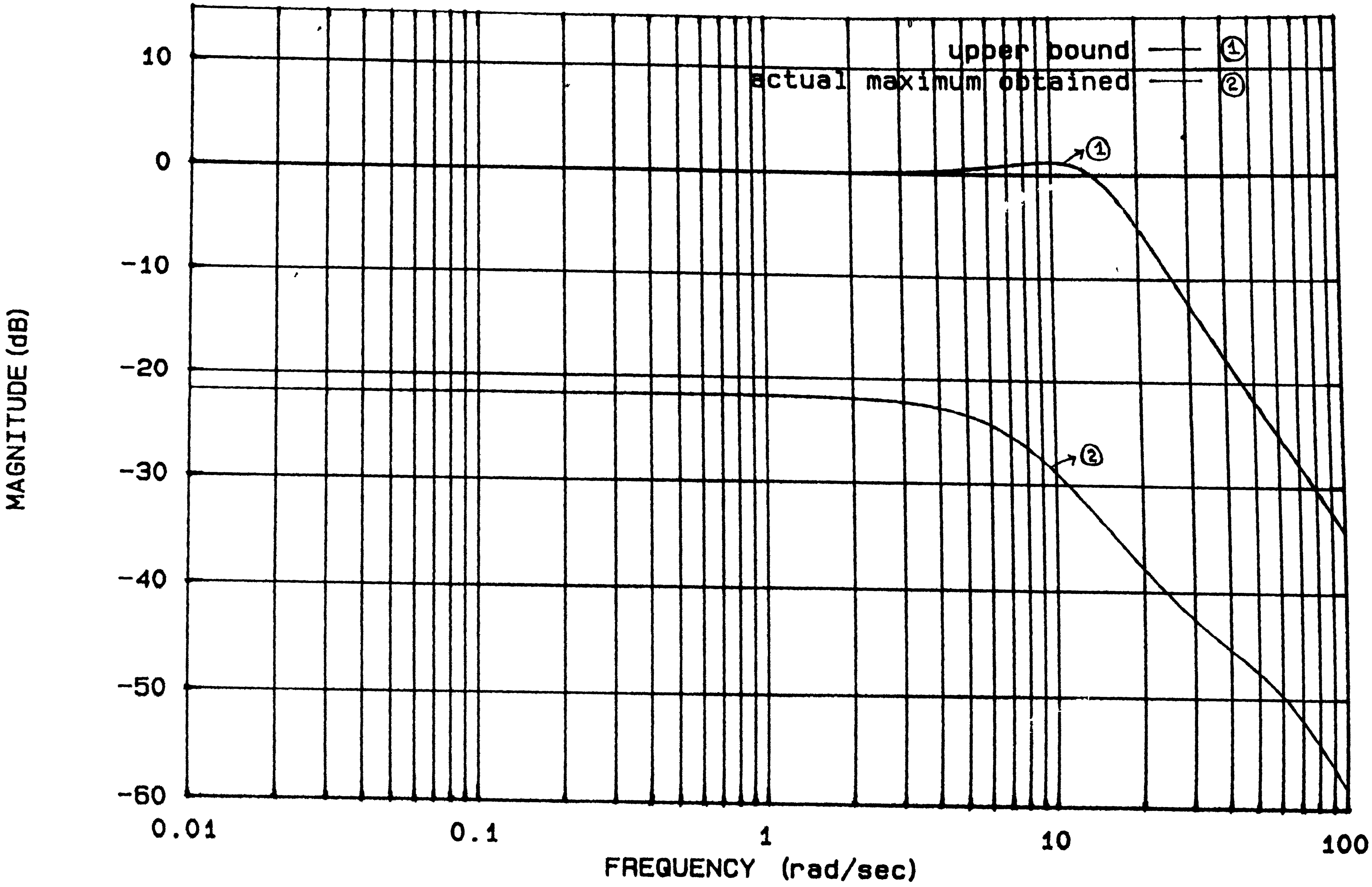
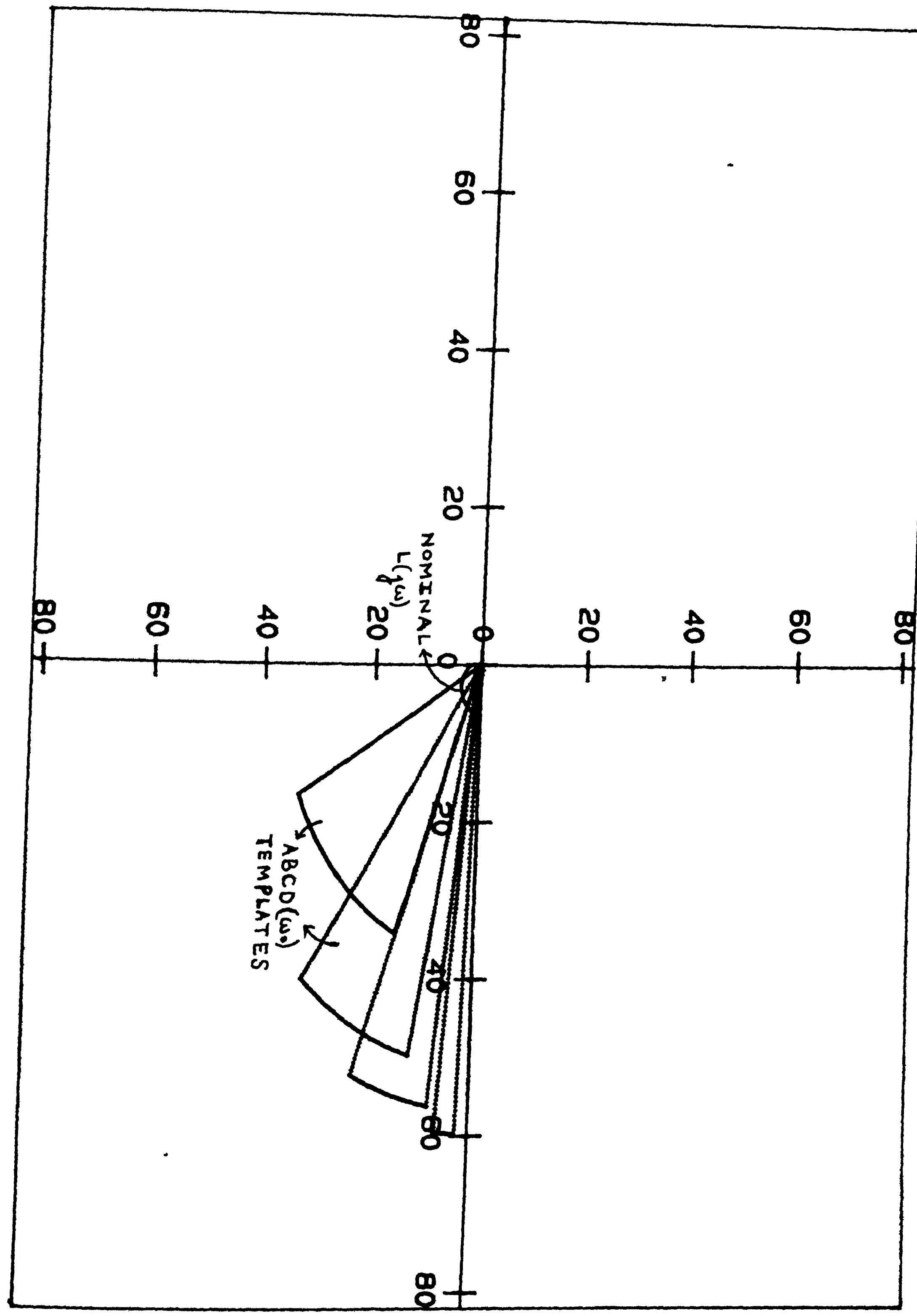


FIGURE 5.12 IMPULSE RESPONSE --- EXAMPLE 4

FIG. 5.13 BODE MAGNITUDE PLOT OF ALLOWED AND OBTAINED DISTURBANCE - EXAMPLE 4



IMAG

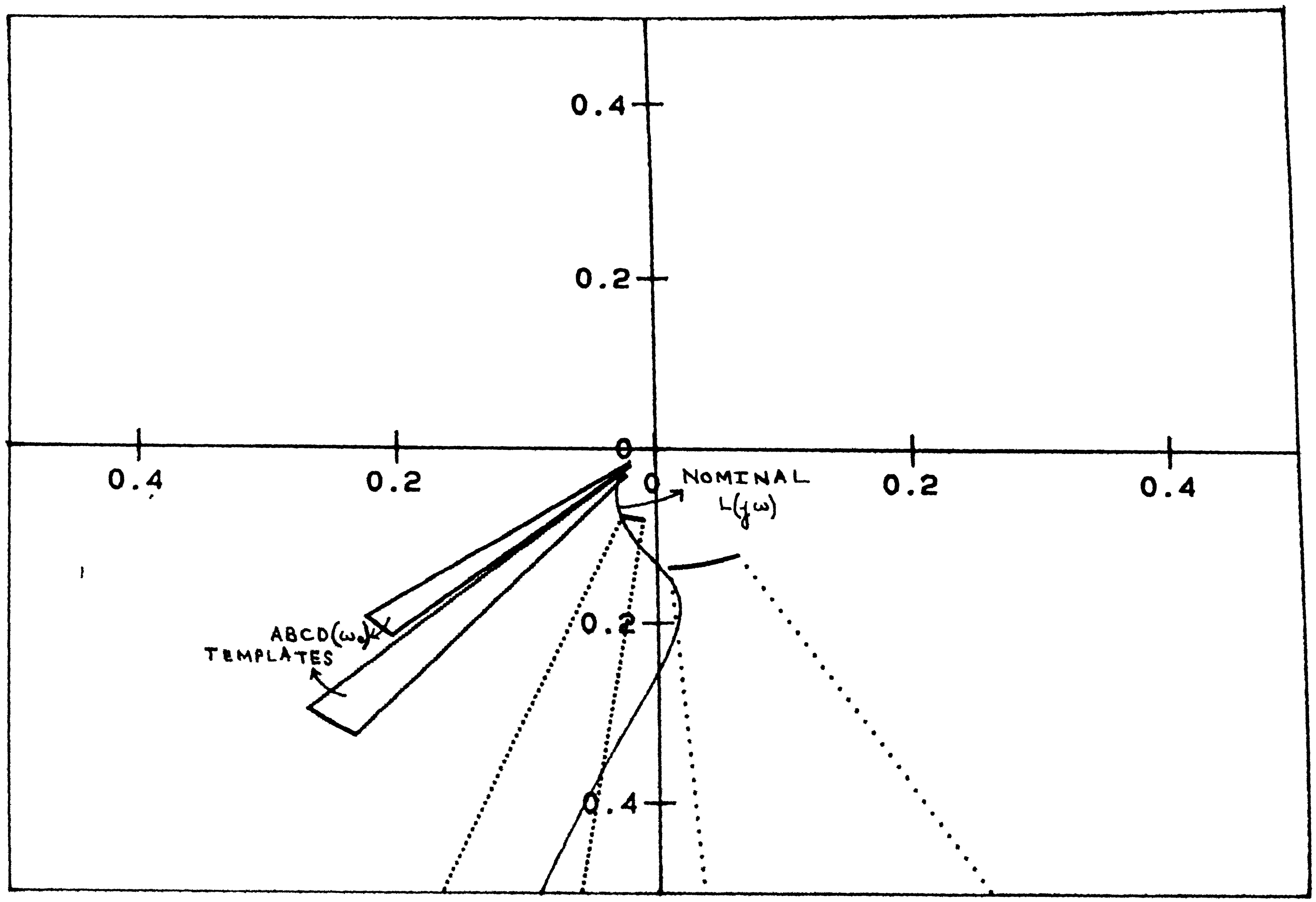


REAL

FIGURE 5.14 LOW FREQUENCY POLAR PLOT OF OPEN LOOP GAIN --- EXAMPLE 4

FIGURE 5.15 HIGH FREQUENCY POLAR PLOT OF OPEN LOOP GAIN - EXAMPLE 4

IMAG



REAL

FIGURE 5.16 IMPULSE RESPONSE --- EXAMPLE 5

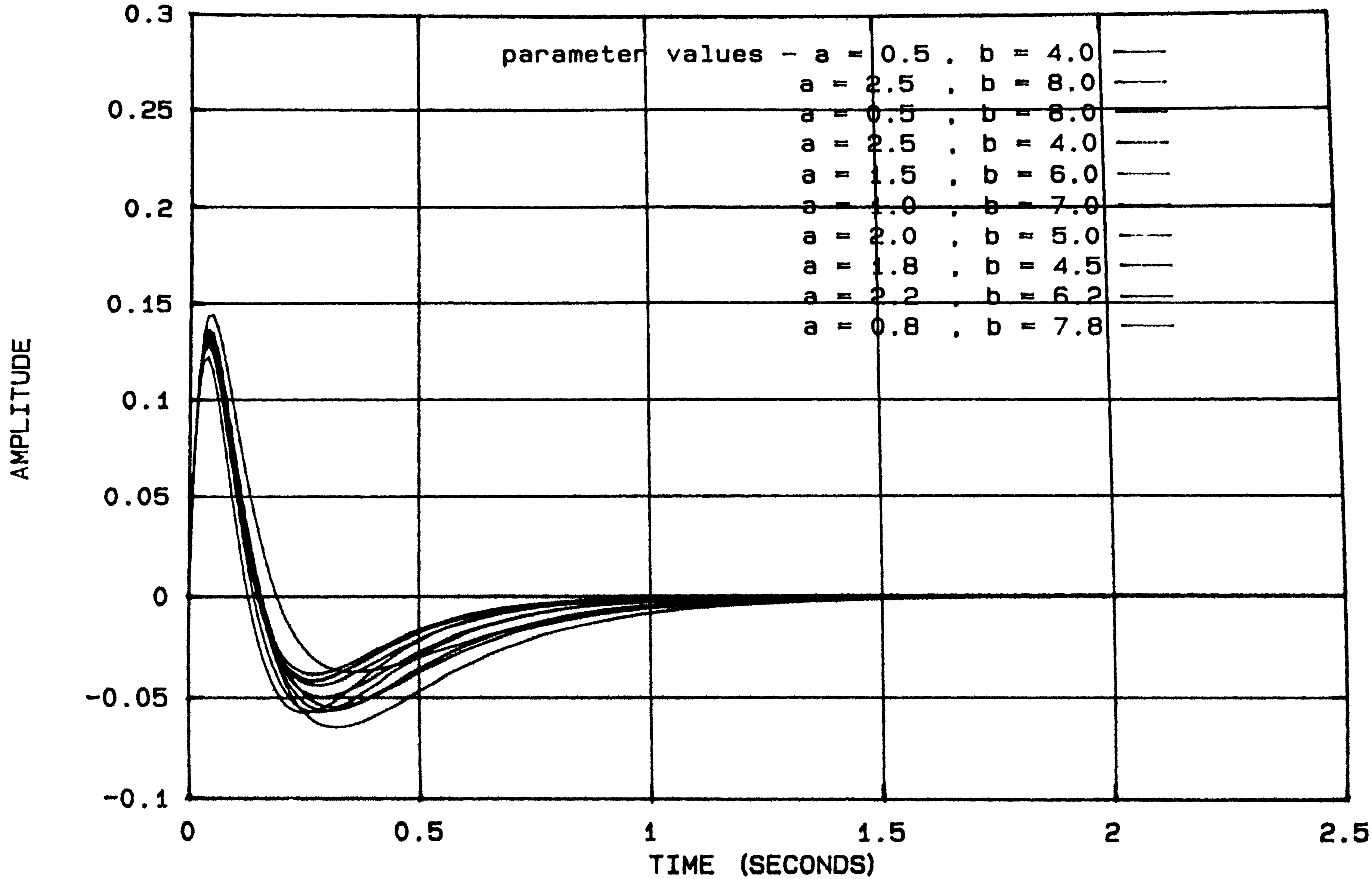


FIG. 5.17 BODE MAGNITUDE PLOT OF ALLOWED AND OBTAINED DISTURBANCE - EXAMPLE 5

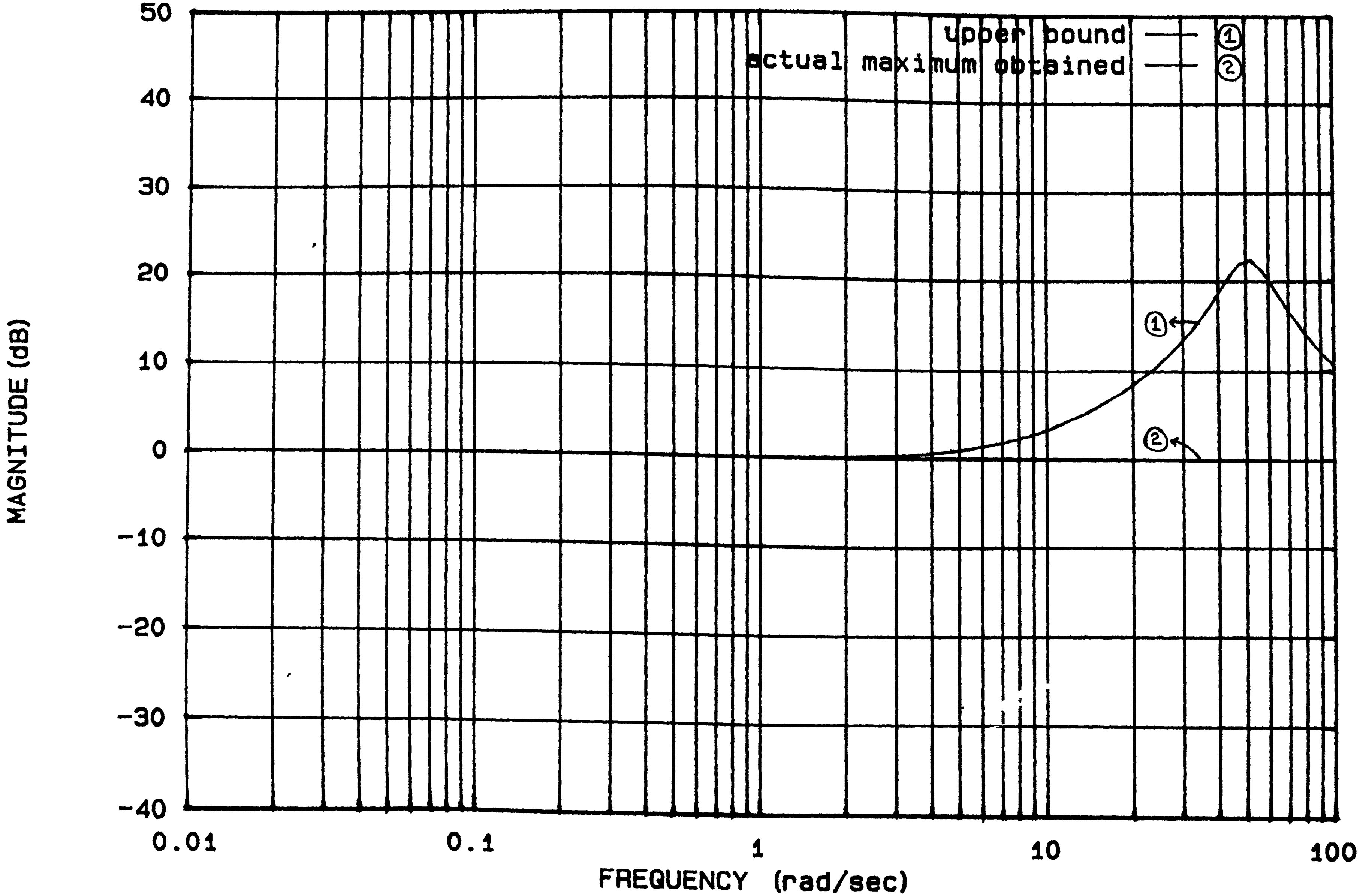
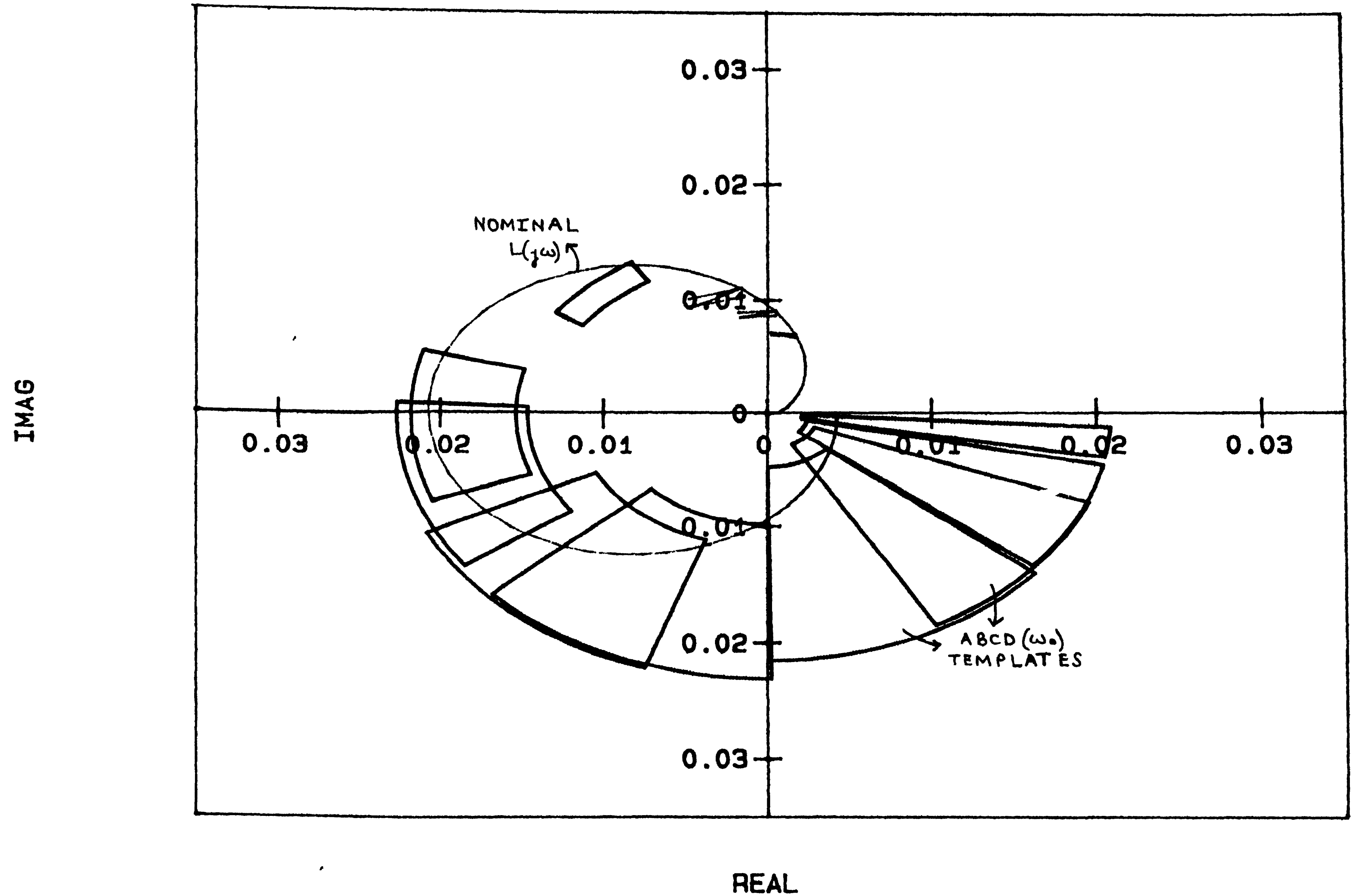


FIGURE 5.18 POLAR PLOT OF OPEN LOOP GAIN --- EXAMPLE 5



CHAPTER 6

CONCLUSIONS

An issue in feedback design, which has been a subject of intense research in the last few years, is that of ensuring system performance in the face of large variations in the values of the system parameters. That QFT is quite effective and easy to apply in solving this design problem, at least for the SISO case, has been demonstrated in this thesis through the solution of five varied examples (refer chapter 5). A software package, quite general enough to handle arbitrary polynomially uncertain systems (refer section 4.4, chapter 4), has been developed to provide computer-aided QFT design. This software has the following unique features -

(1) The use of the Nichols chart, as advocated in classical QFT [JAY93], is dispensed off with. The resulting saving in computational labour is phenomenal.

(2) Dependence on graphical utilities, which is very high in classical QFT [BAI91], [BAR93], is greatly reduced, when using this software (refer appendix A). It also reveals QFT to be essentially a natural extension of the traditional loop shaping based design of the kind found in e.g.[BOW58]. These factors help in enhancing the user friendly aspect of QFT. Anyone, with a basic knowledge of classical loop shaping, can quickly and effectively learn to grapple, using this software, with realistic

design problems involving large parametric uncertainty.

6.1 Beyond single loop LTI SISO QFT design

Though only continuous time, LTI SISO systems in a single loop feedback configuration (refer fig. 2.1), have been considered in this thesis, it may well be noted that QFT has been used to handle the following types of systems as well -

(1) Multiple loop continuous time, LTI SISO systems.

(2) Discrete time LTI SISO systems.

(3) MIMO LTI systems.

(4) Time varying systems.

(5) Nonlinear systems.

An exhaustive survey of the literature dealing with these design problems is given in [HOR91]. Related references which have come out fairly recently and which provide for a better and simplified elucidation of the QFT design procedure are - [YAN93], for discrete time SISO LTI systems and [YAN92], [PER93], for MIMO LTI systems.

6.2 Scope for future work

6.2.1 Defining performance specifications

QFT, as described in section 2.2, chapter 2, requires time domain specifications to be converted to the frequency domain. Lack of a theoretical framework under which this conversion can be rigourously effected, forces us to adopt semi-heuristic techniques of the type exemplified in example

2.1, chapter 2. That this could be problematic is shown in examples 5.4 and 5.5, chapter 5. Consequently, this aspect of time domain to frequency domain conversion is one area where research is definitely required. The theory of wavelets [CHI92], needs looking up in this particular regard.

Fairly recently, however, there have been efforts to ensure tracking specifications of the form given in equation 2.4, directly in the time domain [BAR93]. This approach, known as time domain QFT, however, is in its infancy and provides another area for further research.

6.2.2 Usability of the software developed in this thesis

The use of two NAG routines (refer appendix A) by the software developed in this thesis, limits its use to computer systems loaded with the NAG math library. Also, a convenient and transparent interface to MATLAB has not been provided here. Thus there is scope for improvement of this software to increase its portability and ease of use.

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APPENDIX A

Here, a brief description of the software developed in this thesis is provided with the help of an example.

A.1 General description of the QFT design software

A.1.1 Uncertain system specification

The uncertain system should be specified in the form given in equation 2.2, chapter 2. Here, each coefficient $a_i(q)$ is assumed to be some multivariable polynomial in the uncertain parameter vector **q**, with real coefficients, i.e.,

$$a_i(\mathbf{q}) = \sum_{j=0}^{r} \alpha_j \, t_j^{a_i} \qquad , \qquad \alpha_j \in \mathbb{R},$$

where $$t_j^{a_i} = \prod_{k=0}^{n-1} (q_k)^{s_{jk}} \quad ,$$

where "n" is the number of uncertain real system parameters $q_k \in [q_k^- , q_k^+] \subseteq \mathbb{R}$, while "r" and "$s_{jk}$" are some non-negative integers.

Each "$t_j^{a_i}$" is known as a "term" in the coefficient $a_i(q)$.

A.1.2 Specification of the trial frequencies' set Ω

The trial frequencies' set Ω can be specified to be any arbitrary finite set of real frequencies, in the units of radians per second. Provision is also made to specify a set of logspaced frequencies (see definition 5.1, chapter 5) in the interval $[10^{d_1}, 10^{d_2}]$, where d_1 and d_2 are integers with $d_1 \leq d_2$.

A.1.3 Problem type specification

The following types of design problems can be handled, one at a time, by this software -

TYPE 1 The performance specification of **equation 2.5(a)**, chapter 2, is handled here and involves synthesising a G(s) and, if necessary, an F(s). Examples 5.1 and 5.2, chapter 5, are of this problem type. The performance admissibility of $|G(j\omega)|$ at each trial frequency $\omega \in \Omega$ (see section 3.2, chapter 3) is tested. If inadmissible a bound for $|G(j\omega)|$ is found. This bound will be a lower one for a non-zero $T^{l}_{yr}(s)$ and upper one for $T^{l}_{yr}(s) \equiv 0$. A $T^{l}_{yr}(s) \equiv 0$ allows one to have a performance specification of the form

$$|T_{yr}(j\omega)| \le K/(\omega^n) \ , \quad \forall\omega \ge \omega_h$$

where ω_h is some high frequency value and n is some integer with $K > 0$.

TYPE 2 A disturbance attenuation specification of **equation 2.5(b)**, chapter 2 (see figure 2.1, chapter 2, and example 5.4, chapter 5) falls under this design problem type. It involves synthesis of G(s) alone, as the pre-filter F(s) is not needed here. A lower bound for $|G(j\omega)|$ is found, if G(s) is found inadmissible at the trial frequency $\omega \in \Omega$.

TYPE 3 A disturbance attenuation specification of **equation 2.5(c)**, chapter 2 (see figure 2.1, chapter 2 and example 5.5, chapter 5) or a **sensitivity specification** of the kind given in example 5.3, chapter 5, falls under this design problem type. It involves, as in a type 2

problem, synthesis of G(s) alone, as the pre-filter is not needed. A lower bound for $|G(j\omega)|$ is found, if G(s) is found inadmissible at the trial frequency $\omega \in \Omega$.

A.1.4 Testing closed loop stability

For a performance admissible G(s), at each trial frequency $\omega \in \Omega$, the loop gain extrema (i.e. of the magnitude and phase of $G(j\omega)$) are found. For a linear affine uncertain system (definition 4.3.2, section 4.3, chapter 4), the restricted region method (section 4.2, chapter 4) can be used. For any other arbitrary polynomially uncertain system (definition 4.4.1, section 4.4, chapter 4) use the grid method (section 4.2, chapter 4). These extrema can be then used to visualise a rough open loop polar plot involving the ABCD(ω_o) - type templates (section 4.1, chapter 4, and e.g. of the form given in figure 5.3, chapter 5). Applying the Nyquist criteria to this fuzzy open loop polar plot, stability can then be determined *without resort to the use of any computer graphics utility.*

A.1.5 Operational details of the software developed

The programming has been done in C and Fortran, with a transparent interface between the C and Fortran modules. The Fortran module was necessitated due to the use of two NAG routines viz. [NAG90] -

(1) E04JAF - a quasi Newton optimisation algorithm, on which the restricted region method is based.

(2) C02AFF - an algorithm to find all the roots of a complex polynomial, which aids phase computation.

Actual operational details of the software developed, are provided in a help file called "QFT_help".

A.2 Example

Consider the design problem given in example 5.3, chapter 5. This is a type-3 problem (section A.1.3) with the following details, which were input, aided by prompting of the computer program -

Uncertain system - $$P(s) = \frac{(s + a)}{(s + 3)(s + 10)(s + b)}$$

$$= \frac{(s + a)}{s^3 + (13 + b)s^2 + (30 + 13b)s + 30b}$$

Bounds on the - uncertain parameters

$a \in [0.5, 2.5]$

$b \in [4.0, 8.0]$

Problem type - This is a type 3 problem with the upper bound

$T^u_{ydz}(j\omega) = 0.1$, $\forall\omega \in [0.0, 0.3]$, and

$T^u_{ydz}(j\omega) = 100$, $\forall\omega > 0.3$

Trial frequencies set - Ω was chosen to be a set of 100 logspaced frequencies in the range $[10^{-2}, 10^{2}]$ rad/sec.

Candidate G(s) - A candidate G(s) was specified to be

$$G(s) = \frac{15.12 \times 10^5}{(s + 350)}$$

This candidate G(s) eventually turned out to be

a solution for the given design problem. To see the ensuring of closed loop stability by this G(s), the loop gain extrema are provided in the following table, viz. table A.1.

FREQUENCY (rad/sec)	MINIMUM LOOP MAGNITUDE (dB)	MAXIMUM LOOP MAGNITUDE (dB)	MINIMUM LOOP PHASE (degrees)	MAXIMUM LOOP PHASE (degrees)
0.010000	19.086528	39.084840	-0.163975	0.824225
0.010975	19.086871	39.084838	-0.179963	0.904553
0.012045	19.087284	39.084835	-0.197509	0.992696
0.013219	19.087781	39.084832	-0.216759	1.089393
0.014508	19.088380	39.084829	-0.237896	1.195542
0.015923	19.089102	39.084824	-0.261099	1.312042
0.017475	19.089971	39.084819	-0.286548	1.439788
0.019179	19.091017	39.084812	-0.314490	1.580001
0.021049	19.092277	39.084805	-0.345153	1.733815
0.023101	19.093794	39.084795	-0.378802	1.902522
0.025354	19.095621	39.084784	-0.415746	2.087651
0.027826	19.097820	39.084771	-0.456281	2.290643
0.030539	19.100468	39.084755	-0.500769	2.513245
0.033516	19.103654	39.084735	-0.549587	2.757274
0.036784	19.107488	39.084711	-0.603176	3.024847
0.040370	19.112102	39.084683	-0.661981	3.318047
0.044366	19.117653	39.084649	-0.726525	3.639325
0.048626	19.124328	39.084608	-0.797368	3.991236
0.053367	19.132355	39.084558	-0.875115	4.376505
0.058570	19.142001	39.084498	-0.960442	4.798082
0.064281	19.153591	39.084426	-1.054101	5.259197
0.070548	19.167505	39.084339	-1.156881	5.763069
0.077426	19.184202	39.084235	-1.269685	6.313262
0.084975	19.204223	39.084108	-1.393499	6.913456
0.093260	19.228208	39.083957	-1.529391	7.567354

Table A.1 (continued on the next page)

FREQUENCY (rad/sec)	MINIMUM LOOP MAGNITUDE (dB)	MAXIMUM LOOP MAGNITUDE (dB)	MINIMUM LOOP PHASE (degrees)	MAXIMUM LOOP PHASE (degrees)
0.102353	19.256911	39.083773	-1.678545	8.278736
0.112332	19.291214	39.083553	-1.842244	9.051234
0.123285	19.332152	39.083286	-2.021935	9.888473
0.135305	19.380914	39.082966	-2.219151	10.793459
0.148497	19.438877	39.082578	-2.435623	11.768852
0.162975	19.507603	39.082112	-2.673231	12.816407
0.178865	19.588857	39.081548	-2.934059	13.936818
0.196304	19.684595	39.080868	-3.220372	15.129205
0.215443	19.796956	39.080046	-3.534675	16.390829
0.236449	19.928244	39.079053	-3.879741	17.716690
0.259502	20.080848	39.077853	-4.258567	19.098797
0.284804	20.257226	39.076400	-4.674528	20.526196
0.312572	20.459752	39.074640	-5.131258	21.984107
0.343047	20.690656	39.072506	-5.632812	23.454113
0.376494	20.951883	39.069917	-6.183664	24.913982
0.413201	21.244936	39.066769	-6.788704	26.337709
0.453488	21.570794	39.062936	-7.453392	27.696220
0.497702	21.929745	39.058261	-8.183682	28.957730
0.546228	22.321358	39.052548	-8.986220	30.088871
0.599484	22.744382	39.045550	-9.868264	31.055427
0.657933	23.196802	39.036958	-10.837894	31.823537
0.722081	23.675826	39.026379	-11.903977	32.360490
0.792483	24.177970	39.013321	-13.076258	32.635570
0.869749	24.699143	38.997160	-14.365425	32.620679
0.954548	25.234739	38.977105	-15.783119	32.290744
1.047616	25.779736	38.952163	-17.341960	31.623967
1.149757	26.328751	38.921086	-19.055381	30.601996
1.261857	26.876152	38.882319	-20.937602	29.209930
1.384886	27.416084	38.833943	-23.003270	27.436410
1.519911	27.942551	38.773620	-25.267141	25.273592
1.668101	28.449450	38.698546	-27.743470	22.717262
1.830738	28.930629	38.605419	-30.445322	19.766892
2.009233	29.379970	38.490434	-33.383828	16.425585
2.205131	29.791435	38.349321	-36.567155	12.700191

Table A.1 (continued on the next page)

FREQUENCY (rad/sec)	MINIMUM LOOP MAGNITUDE (dB)	MAXIMUM LOOP MAGNITUDE (dB)	MINIMUM LOOP PHASE (degrees)	MAXIMUM LOOP PHASE (degrees)
2.420128	30.159146	38.177417	-39.999667	8.601140
2.656088	30.477464	37.969784	-43.681168	4.142189
2.915053	30.741031	37.721377	-47.606250	-0.659768
3.199267	30.944820	37.427222	-51.764142	-5.785181
3.511192	31.084155	37.082611	-56.138795	-11.212148
3.853529	31.154703	36.683276	-60.709375	-16.916702
4.229243	31.152469	36.225534	-65.451089	-22.873000
4.641589	31.073781	35.706374	-70.336157	-29.053311
5.094138	30.915293	35.123510	-75.334820	-35.427825
5.590810	30.674022	34.475372	-80.416297	-41.964428
6.135907	30.347414	33.761079	-85.549564	-48.628458
6.734151	29.933448	32.980390	-90.703934	-55.382622
7.390722	29.430767	32.133662	-95.849459	-62.187130
8.111308	28.838810	31.221804	-100.957277	-69.000273
8.902151	28.157936	30.246255	-105.999868	-75.779242
9.770100	27.389503	29.208968	-110.951368	-82.481315
10.722672	26.535894	28.112381	-115.787971	-89.065255
11.768120	25.600473	26.959384	-120.488390	-95.492766
12.915497	24.587485	25.753273	-125.034287	-101.729764
14.174742	23.501894	24.497668	-129.410736	-107.747508
15.556761	22.349192	23.196425	-133.606516	-113.523305
17.073526	21.135182	21.853524	-137.614287	-119.040863
18.738174	19.865778	20.472970	-141.430565	-124.290248
20.565123	18.546821	19.058683	-145.055550	-129.267546
22.570197	17.183921	17.614416	-148.492813	-133.974281
24.770764	15.782340	16.143675	-151.748884	-138.416685
27.185882	14.346915	14.649679	-154.832787	-142.604907
29.836472	12.882004	13.135319	-157.755579	-146.552232
32.745492	11.391467	11.603144	-160.529898	-150.274341
35.938137	9.878666	10.055364	-163.169557	-153.788663
39.442061	8.346479	8.493845	-165.689204	-157.113821
43.287613	6.797318	6.920130	-168.104029	-160.269180
47.508102	5.233163	5.335448	-170.429525	-163.274477

Table A.1 (continued on the next page)

FREQUENCY (rad/sec)	MINIMUM LOOP MAGNITUDE (dB)	MAXIMUM LOOP MAGNITUDE (dB)	MINIMUM LOOP PHASE (degrees)	MAXIMUM LOOP PHASE (degrees)
52.140083	3.655588	3.740731	-172.681311	-166.149540
57.223677	2.065784	2.136628	-174.874974	-168.914069
62.802914	0.464590	0.523514	-177.025960	-171.587462
68.926121	-1.147495	-1.098501	-179.149461	-174.188680
75.646333	-2.770294	-2.729566	-181.260324	-176.736129
83.021757	-4.403941	-4.370092	-183.372939	-179.247538
91.116276	-6.048876	-6.020750	-185.501114	-181.739836
100.000000	-7.705841	-7.682473	-187.657931	-184.229000

Table A.1 : Loop gain extrema at all the trial frequencies, for the G(s) constituting a solution for the design problem in example 5.3, chapter 5.

Observe that the fuzzy open loop gain can be perceived to cross the negative real axis at around 75 rad/sec with a maximum loop gain of less than 1.0. This would mean that the open loop gain polar plot does not encircle the critical -1+j0 point. Since the given system is minimum phase, the zero encirclement count would imply a stable closed loop system. This fact is confirmed by the set of stable (i.e. bounded) step responses obtained (see figure 5.11, chapter 5) for a randomly selected set of values of the uncertain parameters "a" and "b".